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# The Extension of Singular Homology on the Category of Soft Topological Spaces

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***Abstract:** In this article we introduce singular cubic homology groups of soft topological spaces, being extension of the relevant homology groups of topological spaces. For this purpose, the concept of soft unit interval is initially introduced. Further, using the concept of soft unit interval, definition the homotopy relation in the category of soft topological spaces is given and the soft homotopic invariance of the entered homology groups is proved.*

## I. INTRODUCTION

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical method have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools. Molodtsov [7] initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Research works in soft set theory and its applications in various fields have been progressing rapidly since Maji et al. ([4], [5]) introduced several operations on soft sets and applied it to decision making problems. The idea of soft topological spaces was first given by M. Shabir, M. Naz [8] and mappings between soft sets were described by P. Majumdar, S.K. Samanta [6]. Later, many researches about soft topological spaces were studied in [1,2,3,9,10,11,12,13,14]. In these studies, the concept of soft point is expressed by different approaches. In the study we use the concept of soft point which was given in [9]. Soft topological spaces and soft continuous mappings form category and this category is extension of category of topological spaces. Also category of fuzzy topological spaces is extension of category of topological spaces. It is known that in research of topological spaces the important place is taken by methods of the algebraic topology [15,16]. Unfortunately, these methods were not widely used in the research of fuzzy topological spaces although there were some articles [17,18,19,20,21]. In our opinion such situation was due to definition of fuzzy as of a unit interval. In category of soft topological spaces, methods of algebraic topology weren't considered yet. In this article we introduce singular cubic homology groups of soft topological spaces, being extension of the relevant homology groups of topological spaces. For this purpose, the concept of soft unit interval is initially introduced. Further, using the concept of soft unit interval, definition the homotopy relation in the category of soft topological spaces is given and the soft homotopic invariance of the introduced homology groups is proved.

## II. PRELIMINARIES

Throughout this paper, $X$ refers to an initial universe, $E$ is the set of all parameters for $X$. We now recall some definitions.

**Definition 2.1. ([7])** Let $X$ be an initial universe set and $E$ be a set of parameters. A pair $(F,E)$ is called a soft set over $X$ if and only if $F$ is a mapping from $E$ into the set of all subsets of the set $X$, i.e., $F:E\to P(X)$, where $P(X)$ is the power set of $X$.

**Definition 2.2. ([5])** The intersection of two soft sets $(F,A)$ and $(G,B)$ over $X$ is the soft set $(H,C)$, where $C=A\cap B$ and $\forall\varepsilon\in C$, $H(\varepsilon)=F(\varepsilon)\cap G(\varepsilon)$. This is denoted by $(F,A)\cap(G,B)=(H,C)$.

**Definition 2.3.** ([5]) The union of two soft sets $(F,A)$ and $(G,B)$ over $X$ is the soft set $(H,C)$, where $C=A\cup B$ and $\forall\varepsilon\in C$,

$$H(\varepsilon)=\begin{cases}F(\varepsilon), \text{ if } \varepsilon\in A-B\\ G(\varepsilon), \text{ if } \varepsilon\in B-A\\ F(\varepsilon)\cup G(\varepsilon), \varepsilon\in A\cap B\end{cases}$$

This is denoted by $(F,A)\cup(G,B)=(H,C)$.

**Definition 2.4. ([5])** Let $(F,A)$ and $(G,B)$ be two soft sets over $X$. Then $(F,A)$ is called a soft subset of $(G,B)$, denoted by $(F,A)\subset(G,B)$, if

(1) $A\subset B$,

(2) $F(a)\subset G(a)$ for each $a\in A$.

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**Definition 2.5. ([5])** A soft set $(F,E)$ over $X$ is said to be an absolute soft set denoted by $\tilde{X}$ if for all $\varepsilon \in E$, $F(\varepsilon)=X$.

**Definition 2.6. ([5])** A soft set $(F,E)$ over $X$ is said to be a null soft set denoted by $\Phi$ if for all $\varepsilon \in E$, $F(\varepsilon)=\varnothing$.

**Definition 2.7. ([8])** Let $\tau$ be a collection of soft sets over $X$. Then $\tau$ is said to be a soft topology on $X$ if

(1) $\Phi, \tilde{X}$ belong to $\tau$
(2) the union of any number of soft sets in $\tau$ belongs to $\tau$,
(3) the intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X,\tau,E)$ is called a soft topological space over $X$.

**Definition 2.8. ([9])** Let $(F,E)$ be a soft set over $X$ and $x \in X$. The soft set $(F,E)$ is called a soft point, denoted by $(x_e,E)$, if for the element $e \in E$, $F(e)=\{x\}$ and $F(e')=\varnothing$ for all $e' \in E-\{e\}$. In this case, we say that $(x_e,E)$ is a point of a soft set $(F,E)$.

**Definition 2.9. ([2])** Let $(X,\tau,E)$ and $(Y,\tau',E)$ be two soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. For each soft neighbourhood $(H,E)$ of $(f(x)_e,E)$, if there exists a soft neighbourhood $(F,E)$ of $(x_e,E)$ such that $f((F,E))\subset(H,E)$, then $f$ is said to be soft continuous mapping at $(x_e,E)$.

**Definition 2.10. ([10])** Let $\{(X,T_i)\}_{i\in\Delta}$ be a family of soft topological spaces. Then the initial soft topology on $X\left(=\prod_{i\in\Delta}X_i\right)$ generated by the family $\{(p_q)_i\}_{i\in\Delta}$ is called product soft topology on $X$. (Here $(p_q)_i$ is the soft projection mapping from $X$ to $X_i$, $i\in\Delta$.)

## III. THE MAIN RESULTS

Soft topological spaces and their soft continuous maps form a category. This category is denoted by $Stop$. Each usual topological spaces are obtained soft topological spaces. Actually, let $(X,\tau)$ be any topological spaces. We consider of the single-point parameter set $E=\{*\}$. For every $U\in\tau$, the function $F_U:E\to P(X)$ is defined as $F_U(*)=U$. The family $\tau^*=\{F_U\}_U\subset\tau$ forms a soft topology over $X$. Thus, every topological space can be handled a soft topological space. So, the category of topological spaces is a subcategory the category of soft topological spaces. Unless otherwise stated $E=N$ will be assumed to be a set of parameters and the set of rational numbers on the closed interval $I=[0,1]$ will be considered as the set $\{r_n\}$. If for all $n\in N$ and for all $\varepsilon>0$ we define the soft set $F_\varepsilon:N\to P(I)$ as $F_\varepsilon(n)=(r_n-\varepsilon,r_n+\varepsilon)\cap I$. Then the family $\mathrm{B}=\{(F_\varepsilon,N)\}_{\varepsilon>0}$ is a soft base of soft topology on $I$ and $\tau_I$ is called a soft topology generated by $\mathrm{B}$.

**Definition 3.1.** A soft topological space $(I,\tau_I,N)$ is called a soft unit interval.

**Definition 3.2.** Let $(X,\tau,E)$ be a soft topological space. Then a soft singular $n-$cube in $(X,\tau,E)$ is a soft continuous map

$$(T,\psi):(I^n,\tau_{I^n},N^n)\to(X,\tau,E),$$

where $T:I^n\to X$, $\psi:N\to E$. Here $I^n$ is the topological product of soft topological space $(I,\tau_I,N)$ and $\tau_{I^n}$ denotes the soft product topology [10].

$S_n(X,\tau,E)$ denotes the set of all soft singular $n-$cubes in $(X,\tau,E)$.

**Definition 3.3.** A soft singular $n-$cube $(T,\psi)\in S_n(X,\tau,E)$ $(n\geq 1)$ is called degenerate if only if the mapping $(T,\psi)$ does not depend on one of the coordinate values.

**Definition 3.4.** Let $(X,\tau,E)$ be a soft topological space, and $Z$ be the (usual) additive group of integers.

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For $n \geq 1$, $Q_n\left[(X,\tau,E),Z\right]$ denotes the free abelian group generated by the set $S_n(X,\tau,E)$ of all soft singular $n-$cubes in $(X,\tau,E)$.

For $n \geq 1$, let $D_n\left[(X,\tau,E),Z\right]$ denote the subgroup of $Q_n\left[(X,\tau,E),Z\right]$ generated by all degenerate soft singular $n-$cubes, and $D_0\left[(X,\tau,E),Z\right]=\{0\}$.

Then we can define

$$C_n\left[(X,\tau,E),Z\right]=\frac{Q_n\left[(X,\tau,E),Z\right]}{D_n\left[(X,\tau,E),Z\right]},\quad n>0$$

and called it the group of singular $n-$chains in $(X,\tau,E)$.

For all $1 \leq i \leq n$ and $k \in N$, we define the following soft mappings

$$\left(A_i^{0(k)},\varphi_i^{(k)}\right),\left(A_i^{1(k)},\varphi_i^{(k)}\right):\left(I^{n-1},\tau_{I^{n-1}},N^{n-1}\right)\to\left(I^n,\tau_{I^n},N^n\right)$$

by the formula:

$$\varphi_i^{(k)}(j_1,\ldots,j_{n-1})=(j_1,\ldots,j_{i-1},k,j_i,\ldots,j_{n-1})$$

$$A_i^{0(k)}\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-1}}\right)=\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{i-1}},\tilde{0}_k,\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-1}}\right)$$

$$A_i^{1(k)}\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-1}}\right)=\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{i-1}},\tilde{1}_k,\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-1}}\right).$$

It is clear that the soft mappings $\left(A_i^{0(k)},\varphi_i^{(k)}\right),\left(A_i^{1(k)},\varphi_i^{(k)}\right)$ are soft continuous [2].

**Definition 3.5.** Let $(T,\psi)\in S_n(X,\tau,E)$, $n \geq 1$. For $i=1,2,\ldots,n$,

$\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)=(T,\psi)\circ\left(A_i^{0(k)},\varphi_i^{(k)}\right)$ and

$\left(A_i^{1(k)},\varphi_i^{(k)}\right)(T,\psi)=(T,\psi)\circ\left(A_i^{1(k)},\varphi_i^{(k)}\right)\in S_{n-1}(X,\tau,E)$

are called the $i-$th lower face and $i-$th upper face of $(T,\psi)$, respectively.

**Proposition 3.6.** Let $(T,\psi)\in S_n(X,\tau,E)$, and $1 \leq i \leq j \leq n$. Then

(a)

$$\left(A_i^{0(k)},\varphi_i^{(k)}\right)\circ\left(A_s^{0(l)},\varphi_s^{(l)}\right)(T,\psi)=\left(A_{s-1}^{0(l)},\varphi_{s-1}^{(l)}\right)\circ\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)$$

(b)

$$\left(A_i^{1(k)},\varphi_i^{(k)}\right)\circ\left(A_s^{1(l)},\varphi_s^{(l)}\right)(T,\psi)=\left(A_{s-1}^{1(l)},\varphi_{s-1}^{(l)}\right)\circ\left(A_i^{1(k)},\varphi_i^{(k)}\right)(T$$

(c) $\left(A_i^{0(k)},\varphi_i^{(k)}\right)\circ\left(A_s^{1(l)},\varphi_s^{(l)}\right)(T,\psi)=\left(A_{s-1}^{1(l)},\varphi_{s-1}^{(l)}\right)\circ\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)$

(d) $\left(A_i^{1(k)},\varphi_i^{(k)}\right)\circ\left(A_s^{0(l)},\varphi_s^{(l)}\right)(T,\psi)=\left(A_{s-1}^{0(l)},\varphi_{s-1}^{(l)}\right)\circ\left(A_i^{1(k)},\varphi_i^{(k)}\right)(T,\psi)$.

**Proof. (a)** For any $(T,\psi)\in S_n(X,\tau,E)$, we have

$$\left(A_i^{0(k)},\varphi_i^{(k)}\right)\left(A_s^{0(l)},\varphi_s^{(l)}\right)(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-2}}\right)=\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)\left(A_s^{0(l)},\varphi_s^{(l)}\right)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-2}}\right)$$

$$=(T,\psi)\left(A_i^{0(k)},\varphi_i^{(k)}\right)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{s-1}},\tilde{0}_l,\tilde{x}_{j_s},\ldots,\tilde{x}_{j_{n-2}}\right)$$

$$=(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{i-1}},\tilde{0}_k,\tilde{x}_{j_i},\ldots,\tilde{x}_{j_{s-1}},\tilde{0}_l,\tilde{x}_{j_s},\ldots,\tilde{x}_{j_{n-2}}\right)$$

$$\left(A_{s-1}^{0(l)},\varphi_{s-1}^{(l)}\right)\circ\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-2}}\right)=\left(A_{s-1}^{0(l)},\varphi_{s-1}^{(l)}\right)(T,\psi)\left(A_i^{0(k)},\varphi_i^{(k)}\right)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{n-2}}\right)$$

$$=(T,\psi)\left(A_{s-1}^{0(l)},\varphi_{s-1}^{(l)}\right)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{i-1}},\tilde{0}_k,\tilde{x}_{j_i},\ldots,\tilde{x}_{j_{n-2}}\right)$$

$$=(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{i-1}},\tilde{0}_k,\tilde{x}_{j_i},\ldots,\tilde{x}_{j_{s-1}},\tilde{0}_l,\tilde{x}_{j_s},\ldots,\tilde{x}_{j_{n-2}}\right)$$

Hence (a) holds. Similarly, (b), (c), (d) are easily checked.

**Definition 3.7.** Let $(X,\tau,E)$ be a soft topological spaces. We define a homomorphism

$$\partial_n:Q_n\left[(X,\tau,E),Z\right]\to Q_{n-1}\left[(X,\tau,E),Z\right],$$

such that for $(T,\psi)\in S_n(X,\tau,E)$,

$$\partial_n(T,\psi)=\sum_{i=1}^{n}(-1)^i\left[\left(A_i^{1(k)},\varphi_i^{(k)}\right)(T,\psi)-\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi\right.$$

.

**Lemma 3.8.** Let $n>1$, we have $\partial_{n-1}\circ\partial_n=0$.

**Proof.** We show only that for any $(T,\psi)\in S_n(X,\tau,E)$,

$$\partial_{n-1}\circ\partial_n(T,\psi)=\tilde{0}_k.$$

Indeed, since

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$$\partial_{n-1}\circ\partial_n(T,\psi)=\partial_{n-1}\sum_{i=1}^{n}(-1)^i\left[\left(A_i^{1(k)},\varphi_i^{(k)}\right)(T,\psi)-\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)\right]$$

$$=\sum_{i=1}^{n}(-1)^i\,\partial_{n-1}\left[\left(A_i^{1(k)},\varphi_i^{(k)}\right)(T,\psi)-\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)\right]$$

$$=\sum_{i=1}^{n}(-1)^i\left\{\sum_{s=1}^{n-1}(-1)^s\left[\begin{array}{l}\left(A_s^{1(k)},\varphi_s^{(k)}\right)\left(A_i^{1(k)},\varphi_i^{(k)}\right)-\left(A_s^{0(k)},\varphi_s^{(k)}\right)\left(A_i^{1(k)},\varphi_i^{(k)}\right)-\left(A_s^{1(k)},\varphi_s^{(k)}\right)\\ \left(A_s^{1(k)},\varphi_s^{(k)}\right)\left(A_i^{0(k)},\varphi_i^{(k)}\right)+\left(A_s^{0(k)},\varphi_s^{(k)}\right)\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)\end{array}\right]\right.$$

$$=\sum_{i=1}^{n}\sum_{s=1}^{n-1}(-1)^{i+s}\left\{\sum_{s=1}^{n-1}(-1)^s\left[\begin{array}{l}\left(A_s^{1(k)},\varphi_s^{(k)}\right)\left(A_i^{1(k)},\varphi_i^{(k)}\right)-\left(A_s^{0(k)},\varphi_s^{(k)}\right)\left(A_i^{1(k)},\varphi_i^{(k)}\right)-\\ \left(A_s^{1(k)},\varphi_s^{(k)}\right)\left(A_i^{0(k)},\varphi_i^{(k)}\right)+\left(A_s^{0(k)},\varphi_s^{(k)}\right)\left(A_i^{0(k)},\varphi_i^{(k)}\right)(T,\psi)\end{array}\right]\right.$$

and

$$\left(A_i^{\varepsilon(k)},\varphi_i^{(k)}\right)\left(A_s^{\eta(k)},\varphi_s^{(k)}\right)(T,\psi)=\left(A_{s-1}^{\eta(k)},\varphi_{s-1}^{(k)}\right)\left(A_i^{\varepsilon(k)},\varphi_i^{(k)}\right)(T,\psi)$$

for $1\le i\le s\le n$, where $\varepsilon$ and $\eta$ may be either 0 or 1, if we denote

$$\sum_{i=1}^{n}\sum_{s=1}^{n-1}(-1)^{i+j}\left(A_i^{\varepsilon(k)},\varphi_i^{(k)}\right)\left(A_s^{\eta(k)},\varphi_s^{(k)}\right)(T,\psi)=\sum_{i<s}+\sum_{s\le i},$$

then

$$\sum_{s\le i}(-1)^{i+s}\left(A_s^{\eta(k)},\varphi_s^{(k)}\right)\left(A_i^{\varepsilon(k)},\varphi_i^{(k)}\right)(T,\psi)=\sum_{s<i}(-1)^{i+s+1}\left(A_s^{\eta(k)},\varphi_s^{(k)}\right)\left(A_{i-1}^{\varepsilon(k)},\varphi_{i-1}^{(k)}\right)(T,\psi)$$

$$=\sum_{s<i}(-1)^{i+s+1}\left(A_i^{\varepsilon(k)},\varphi_i^{(k)}\right)\left(A_s^{\eta(k)},\varphi_s^{(k)}\right)(T,\psi)$$

$$=\sum_{i<s}(-1)^{i+s+1}\left(A_i^{\varepsilon(k)},\varphi_i^{(k)}\right)\left(A_s^{\eta(k)},\varphi_s^{(k)}\right)(T,\psi)$$

Hence, $\partial_{n-1}\circ\partial_n(T,\psi)=\tilde{0}_k$ is obtained.

**Proposition 3.9.** Let $n\ge 1$, then $\partial_n(D_n[(X,\tau,E),Z])\subset D_{n-1}[(X,\tau,E),Z]$.

**Proof.** Let $(T,\psi)\in D_n[(X,\tau,E),Z]$. Then we shall prove only that for $(T,\psi)\in D_n[(X,\tau,E),Z]$, $\partial_n(T,\psi)\in D_{n-1}[(X,\tau,E),Z]$.

Indeed, suppose that $(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_i},\ldots,\tilde{x}_{j_n}\right)$ does not depend on $\tilde{x}_{j_i}$, $1\le i\le n$. Then for $i<s\le n$, $\tilde{x}_{j_i}$, $\tilde{y}_{j_i}$

$$(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_i},\ldots,\tilde{x}_{j_n}\right)=\left(A_s^{\varepsilon(k)},\varphi_s^{(k)}\right)(T,\psi)\left(\ldots\right.$$

is satisfied. For $s<i\le n$ and $\tilde{x}_{j_{i-1}}$, $\tilde{y}_{j_{i-1}}$,

$$(T,\psi)\left(\tilde{x}_{j_1},\ldots,\tilde{x}_{j_{i-1}},\ldots,\tilde{x}_{j_n}\right)=\left(A_s^{\varepsilon(k)},\varphi_s^{(k)}\right)(T,\psi)\left(\ldots\right.$$

Thus $\left(A_s^{1(k)},\varphi_s^{(k)}\right)(T,\psi)=\left(A_s^{0(k)},\varphi_s^{(k)}\right)(T,\psi)$.

Hence $\partial_n(T,\psi)\in D_{n-1}[(X,\tau,E),Z]$ is obtained.

As a consequence of Proposition 3.9, $\partial_n$ induces a homomorphism

$$\partial_{*n}:C_n[(X,\tau,E),Z]\to C_{n-1}[(X,\tau,E),Z],\quad n\ge 1.$$

Thus for the soft topological space $(X,\tau,E)$, the following chain complexes of groups

$$\left\{C_n[(X,\tau,E),Z],\partial_{*n}:C_n[(X,\tau,E),Z]\to C_{n-1}[(X,\tau,E\ldots\right.$$

is obtained.

Now, let $(f,g):(X,\tau,E)\to(Y,\tau',E')$ be a soft continuous mapping of soft topological spaces. We define a homomorphism

$$(f,g)_{*n}:S_n(X,\tau,E)\to S_n(Y,\tau',E')$$

such that for any $(T,\psi)\in S_n(X,\tau,E)$,

$$(f,g)_{*n}(T,\psi)=(f,g)\circ(T,\psi)=(f\circ T,g\circ\psi)\in S_n(Y,\tau',E\ldots$$

.

The following proposition can be easily checked.

**Proposition 3.10.** For $n\ge 0$, $(f,g)_{*n}\{D_n[(X,\tau,E),Z]\}\subset D_n[(Y,\tau',E'),Z]$.

From the Proposition 3.10, $(f,g)_{*n}$ induces a homomorphism

$$(f,g)_{*n}:C_n[(X,\tau,E),Z]\to C_n[(Y,\tau',E'),Z].$$

**Proposition 3.11.** Let $(X,\tau,E)$ be a soft topological space. Then the correspondence

$$(X,\tau,E)\mapsto\{C_n[(X,\tau,E),Z]\}$$

$$(f,g)\mapsto\left\{(f,g)_{*n}:C_n[(X,\tau,E),Z]\to C_n[(Y,\tau',E'),Z]\ldots\right.$$

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is a functor from the category of soft topological spaces to the category of chain complexes.

**Proof.** Now we only show that $(f,g)_{*n-1}\partial_n = \partial'_n (f,g)_{*n}$.

$$\partial'_{*n}\left((f,g)_{*n}(T,\psi)\right) = \partial'_{*n}\left[(f,g)(T^n,\psi^n)\right]$$

$$= \sum_{i=1}^{n}(-1)^i\left[(f,g)(T,\psi)A_i^{1(k)} - (f,g)(T,\psi)A_i^{0(k)}\right]$$

$$= (f,g)_{*n-1}\sum_{i=1}^{n}(-1)^i\left[(T^n,\psi^n)A_i^{1(k)} - (T^n,\psi^n)A_i^{0(k)}\right]$$

$$(f,g)_{*n-1}\sum_{i=1}^{n}(-1)^i\left[A_i^{1(k)}(T^n,\psi^n) - A_i^{0(k)}(T^n,\psi^n)\right]$$

$$= (f,g)_{*n-1}\partial_{*n}(T^n,\psi^n).$$

Thus $(f,g)_{*n-1}\partial_n = \partial'_n (f,g)_{*n}$ is satisfied.
Another conditions can be easily obtained.

**Definition 3.12.** Let $(X,\tau,E)$ be a soft topological space. We define as

$$Z_n\left[(X,\tau,E),Z\right] = \ker \partial_{*n}$$

$$B_n\left[(X,\tau,E),Z\right] = \operatorname{Im}\partial_{*n+1}$$

and call them the group of singular $n-$cycles and singular $n-$boundaries in $(X,\tau,E)$, respectively. The group

$$H_n\left[(X,\tau,E),Z\right] = \frac{Z_n\left[(X,\tau,E),Z\right]}{B_n\left[(X,\tau,E),Z\right]}$$

is called the singular $n-$homology group of soft topological space $(X,\tau,E)$.

**Theorem 3.13.** Let $E=\{*\}$ and $(X,\tau)$ be the usual topological space. Then

$$H_n[(X,\tau,E),Z] = H_n(X,Z), \quad n \geq 0$$

where $H_n(X,Z)$ is the $n-$dimensional cubical singular homology group of $X$ (see [16]).

It is clear that for each soft mapping $(f,g):(X,\tau,E)\to(Y,\tau',E')$,

$$(f,g)_{*n}\left(Z_n\left[(X,\tau,E),Z\right]\right) \subset Z_n\left[(Y,\tau',E'),Z\right]$$

and

$$(f,g)_{*n}\left(B_n\left[(X,\tau,E),Z\right]\right) \subset B_n\left[(Y,\tau',E'),Z\right].$$

Then the homomorphism $(f,g)_{*n}$ induces a homomorphism

$$(f,g)_{**n} : H_n\left[(X,\tau,E),Z\right] \to H_n\left[(Y,\tau',E'),Z\right]$$

.

**Theorem 3.14.** The correspondence

$$(X,\tau,E) \mapsto H_n\left[(X,\tau,E),Z\right]$$

$$(f,g) \mapsto \{(f,g)_{**n} : H_n\left[(X,\tau,E),Z\right] \to H_n\left[(Y,\tau',E'),Z\right.$$

is a functor from the category of soft topological spaces to the category of groups.

Now we show that homology functor is homotopic invariant in the category of soft topological spaces. For this, we firstly give soft homotopy relation in the category of soft topological spaces.

**Definition 3.15.** Let $(X,\tau,E)$, $(Y,\tau',E')$ be two soft topological spaces and $(f,\varphi),(g,\psi):(X,\tau,E)\to(Y,\tau',E')$ be two soft continuous mapping. If there exists a soft continuous mapping

$$(F,\phi):(X,\tau,E)\times(I,\tau_I,N)\to(Y,\tau',E')$$

such that

$$(F,\phi)(x_e,0_n) = (f,\varphi)(x_e), \quad (F,\phi)(x_e,1_n) = (g,\psi)(x_e)$$

then we say that $(f,\varphi)$ is soft homotopic to $(g,\psi)$.

Hence we have the following example.

**Example 3.16.** Let $(1_I,1_N),(g,\psi):(I,\tau_I,N)\to(I,\tau_I,N)$ be two soft mappings and define $g(t)=\dfrac{1}{2}, \psi(n)=5$.

If we take $(F,\phi)(x_e,t_n) = (1-t)x_e + t\left(\dfrac{1}{2}\right)_5$, then $(1_I,1_N)$ is soft homotopic to $(g,\psi)$.

In the following, we give some properties of soft homotopy relation.

**Theorem 3.17.** The soft homotopy relation in the category of soft topological spaces is an equivalence relation.

**Proof.** It is obvious that this relation is reflexive and symmetric. We show that soft homotopy relation is transitive.

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Let $(f,\varphi)$ be soft homotopic to $(g,\psi)$ and $(g,\psi)$ be soft homotopic to $(h,\chi)$.

Since $(f,\varphi)$ is soft homotopic to $(g,\psi)$, then there exists a soft continuous mapping $(F,\phi)$ such that $(F,\phi)(x_e,0_n)=(f,\varphi)(x_e)$, $(F,\phi)(x_e,1_n)=(g,\psi)(x_e)$. Since $(g,\psi)$ is soft homotopic to $(h,\chi)$, then there exists a soft continuous mapping $(G,\xi)$ such that $(G,\xi)(x_e,0_n)=(g,\psi)(x_e)$, $(G,\xi)(x_e,1_n)=(h,\chi)(x_e)$. If we take soft continuous mapping $(H,\gamma)$ such that

$$(H,\gamma)(x_e,t_n)=\begin{cases}(F,\phi)(x_e,(2t)_n), & 0\le t\le \frac{1}{2}\\ (G,\xi)(x_e,(2t-1)_n), & \frac{1}{2}\le t\le 1\end{cases}$$

then $(f,\varphi)$ is soft homotopic to $(h,\chi)$.

**Theorem 3.18.** The composition operation in the category of soft topological spaces is invariant with respect to the soft homotopy relation.

Let $(X,\tau,E),(Y,\tau',E')$ be two soft topological spaces. Consider the product soft topological spaces as $(X\times Y,\tau\times\tau',E\times E')$ [10]. A base of soft topology $\tau\times\tau'$ consists of the soft sets $\{(F,E)\times(G,E') \mid (F,E)\in\tau,\ (G,E')\in\tau'\}$. Then for each $(e,e')\in E\times E'$, $(F\times G)(e,e')=F(e)\times G(e')$ is a soft open set in $X\times Y$.

The following proposition easily obtain.

**Proposition 3.19.** Let $(X,\tau,E)$, $(Y,\tau',E')$ be two soft topological spaces. Then $\left(X\times Y,(\tau\times\tau')_{(e,e')}\right)=(X,\tau_e)\times(Y,\tau'_{e'})$ holds.

**Proposition 3.20.** Let $(X,\tau,E)$, $(Y,\tau',E)$ be two soft topological spaces and $(f,\varphi),(g,\varphi):(X,\tau,E)\to(Y,\tau',E)$ be two soft mappings. If $(f,\varphi)$ is soft homotopic to $(g,\varphi)$, then the mappings $f,g:(X,\tau_e)\to\left(Y,\tau'_{\varphi(e)}\right)$ are homotopic for each $e\in E$ [12].

**Proof.** Let the mappings $(f,\varphi),(g,\varphi):(X,\tau,E)\to(Y,\tau',E)$ be soft homotopic and $(F,\phi):(X,\tau,E)\times(I,\tau_I,N)\to(Y,\tau',E)$ be a homotopy between these mappings. For each $e\in E$ and $n\in N$, $F:(X,\tau_e)\times\left(I,(\tau_I)_n\right)\to\left(Y,\tau'_{\varphi(e)}\right)$ is a continuous mapping and the following conditions are satisfied:

$$(F,\phi)(x_e,0_n)=F(x,0)=(f(x))_{\varphi(e)}=f(x),$$
$$(F,\phi)(x_e,1_n)=F(x,1)=(g(x))_{\varphi(e)}=g(x).$$

**Theorem 3.21.** Let $(f,\varphi),(g,\varphi):(X,\tau,E)\to(Y,\tau',E)$ be two soft mappings. If $(f,\varphi)$ is soft homotopic to $(g,\varphi)$, then the morphisms of chain complexes

$$\{(f,\varphi)_{*n}\},\{(g,\varphi)_{*n}\}:\{C_n[(X,\tau,E),Z]\}\to\{C_n[(Y,\tau',E$$

is chain homotopic [15,16].

**Proof.** Let $(F,\phi):(X,\tau,E)\times(I,\tau_I,N)\to(Y,\tau',E)$ be a soft homotopy between the soft mappings $(f,\varphi)$ and $(g,\varphi)$. We will use $(F,\phi)$ to construct a sequence of homomorphisms

$$D_n:C_n[(X,\tau,E),Z]\to C_{n+1}[(Y,\tau',E),Z],$$

$n\ge 0$, such that

$$\partial'_{n+1}D_n+D_{n-1}\partial_n=f_{*n}-g_{*n}.$$

For $n=0$, we define $C_{-1}[(X,\tau,E),Z]=\{0\}=C_{-1}[(Y,\tau',E),Z]$, $\partial_0$ and $D_{-1}:C_{-1}[(X,\tau,E),Z]\to C_0[(Y,\tau',E),Z]$ are the zero homeomorphisms.

First of all we define a sequence of homeomorphisms

$$D_n:C_n[(X,\tau,E),Z]\to C_{n+1}[(Y,\tau',E),Z],$$

$n\ge 1$ as follows. For any $(T,\psi)\in S_n(X,\tau,E)$, define $D_n(T,\psi)\in S_{n+1}(Y,\tau',E)$ by the formula:

$$D_n(T,\psi)\left(x_{i_i},\dots,x_{i_{n+1}}\right)=(F,\phi)\left((T,\psi)\left(x_{i_i},\dots,x_{i_n}\right),x_{i_{n+1}}\right).$$

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We want to compute $\partial'_{n+1}D_n(T,\psi)$. Observe that

$$A^1_{n+1}D_n(T,\psi)=(f,\varphi)_{*n}(T,\psi),\ A^0_{n+1}D_n(T,\psi)=(g,\varphi)_{*n}(T,\psi)$$

$$A^1_iD_n(T,\psi)=D_{n-1}A^1_{i-1}(T,\psi),\ A^0_iD_n(T,\psi)=D_{n-1}A^0_{i-1}(T,\psi)$$

, $2\le i\le n+1$.

Now we have

$$\partial'_{n+1}D_n(T,\psi)=\sum_{i=1}^{n+1}(-1)^i\left[A^1_iD_n(T,\psi)-A^0_iD_n(T,\psi)\right]$$

$$=-\left[(f,\varphi)_{*n}(T,\psi)-(g,\varphi)_{*n}(T,\psi)\right]+\sum_{i=2}^{n+2}(-1)^iD_{n-1}\left[A^1_{i-1}(T,\psi)-A^0_{i-1}(T,\psi)\right]$$

$$=-\left[(f,\varphi)_{*n}(T,\psi)-(g,\varphi)_{*n}(T,\psi)\right]+\sum_{j=1}^{n}(-1)^{j+1}D_{n-1}\left[A^1_j(T,\psi)-A^0_j(T,\psi)\right]$$

$$=-(f,\varphi)_{*n}(T,\psi)-(g,\varphi)_{*n}(T,\psi)+D_{n-1}\partial_n(T,\psi).$$

Therefore for any $(T,\psi)\in S_n(X,\tau,E)$,

$$\partial'_{n+1}D_n(T,\psi)+D_{n-1}\partial_n(T,\psi)=(g,\varphi)_{*n}(T,\psi)-(f,\varphi)_{*n}(T,\psi)$$

. (1)

Now we see that if $(T,\psi)$ is a degenerate soft singular $n-$cube, then $D_n(T,\psi)$ is a degenerate soft $(n+1)-$cube. Hence

$$D_n(D_n(X,\tau,E))\subset D_{n+1}(Y,\tau',E)$$

and therefore $D_n$ induces a homomorphism

$$\tilde{D}_n:C_n\left[(X,\tau,E),Z\right]\to C_{n+1}\left[(Y,\tau',E),Z\right].$$

It follows from (1) that the family of homomorphisms $\left\{\tilde{D}_n\right\}_n$ is a chain homotopy between $\left\{(f,\varphi)_{*n}\right\}$ and $\left\{(g,\varphi)_{*n}\right\}$.

**Corollary 3.22.** Let $(f,\varphi),(g,\varphi):(X,\tau,E)\to(Y,\tau',E)$ be a soft homotopic mappings. Then

$$(f,\varphi)_{**n}=(g,\varphi)_{**n}:H_n\left[(X,\tau,E),Z\right]\to H_n\left[(Y,\tau',E),Z\right]$$

, for all $n\ge 0$.

## IV. CONCLUSION

In this article we introduce singular cubic homology groups of soft topological spaces, being extension of the relevant homology groups of topological spaces. For this purpose, the concept of soft unit interval is initially introduced. Further, using the concept of soft unit interval, definition the homotopy relation in the category of soft topological spaces is given and the soft homotopic invariance of the entered homology groups is proved. As follow up of this paper, we would like to work on the singular homology theory on the category of soft topological spaces, also to prove axiom of the continuity for this theory. We believe that this work will be an incitement for application of methods of algebraic topology in soft topological spaces research and other sciences.

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