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Model Predictive Control for Formation Keeping in an Orbit

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Abstract*— The MPC algorithm concept is widely used in the process industry, but its application in the formation flaying control is rare. This paper presents a MPC algorithm for the formation in an orbit based on the leader-following approach, the linear model implemented in the MPC algorithm is based on the kepler's nonlinear dynamic equation for the relative position. In the suggested control algorithm, a control is to be applied as long as the formation is moving in a prescribed target interval. As the formation leaves that interval, the formation can be left to move naturally after imposing the proper initial states to cause the formation to return back to that interval with approximately the required configuration.

***Index Terms*—Model Predictive control (MPC), Formation keeping in an orbit, nonlinear control,**

I. INTRODUCTION

Formation flying has been identified as an enabling technology for many of NASA's twenty-first-century space and earth science missions. These missions will help to revolutionize our understanding of the origin, environment, and the evolution of planetary systems [1]. The Air Force has also identified formation flying as a key technology for the 21st century.

According to [2], three principal approaches have been developed to coordinate spacecraft in formation. These are leader-following, behavior-based, and virtual structure. In the leader-following (LF) approach, one vehicle is chosen to be the leader while the remaining vehicles are designated as followers. The leader is responsible for achieving the position and attitude goals of the formation mission while the followers are responsible for achieving the formation keeping objectives. In other words, the leader tracks a prescribed trajectory while the followers track the leader position and attitude with a prescribed offset.

Kapila *et. al*. [3] developed a control for low-earth orbit formation flying in a circular orbit. The Clohessy-Wiltshire (C-W) linear dynamic equations are used as a model for the relative position. These equations were originally developed in the context of the spacecraft rendezvous problem. A pulse-based, discrete time feedback control strategy is developed based on full state feedback control, and a linear quadratic regulator (LQR) approach is used to calculate the gains.

Queiroz *et. al* [4] proposed an adaptive nonlinear control for the problem of formation keeping and its stability was proved using Lyapunov approach. The full nonlinear position equations were used for the descriptions of the position of the leader and flower spacecrafts.

McInnes [5] used simple analytic commands to bring a loose ring of satellites into a perfect ring formation with uniform intersatellite spacing in a circular orbit. For each spacecraft, the Keplarian equations of motion are used. A potential function is constructed to maintain the relative orientation of spacecraft. A control law is selected such that this potential function is negative definite.

Abdelkhalik and Alberts [6] developed a controller for the formation in an elliptic orbit based on the leader following approach. The model of the formation flying used for the controller is the Keplarian's nonlinear dynamic equations for the relative position, the inverse dynamic techniques was applied for developing the control low for the formation flaying problem.

Manikoda *et. al* [7] combined the feedback linearization and model predictive control (MPC) to design a controller for space formation keeping and attitude control, the model used for the purpose of designing the MPC controller is based on the assumption of no coupling between each space craft. Moreover, Breger *et. al* [8] used Hill's equations of relative motion in circular orbit that governs the spacecraft to remain inside a specified error box for a formation flying control, the model with an assumed noise were implemented in the MPC algorithm for a formation flying control.

Formation members will, in general, naturally drift away from each other when moving in separate orbits. If they were given proper initial relative velocities that are corresponding to their initial relative positions then they will return to their initial configuration after an orbital period. If formation is required to maintain station keeping over a certain target area then the formation can be controlled during this period only and then the formation will be driven to the appropriate initial states for the free flying period.

The MPC algorithm concept is widely used in the process industry [9, 10, 11], but its application in the formation flaying control is rare. This paper presents a MPC algorithm for the formation in an orbit based on the leader-following approach. In the suggested control algorithm, a control is to be applied as long as the formation is moving in a prescribed target interval. As the formation leaves that interval, the formation can be left to move naturally after imposing the proper initial states to cause the formation to return back to

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that interval with approximately the required configuration. The linear model implemented in the MPC algorithm is the same one that is used by Abdelkhalik and Alberts [6]. The performance of the MPC algorithm is compared with the performance of the nonlinear control technique based on the inverse dynamic to Keplarian's nonlinear dynamics relative motion.

II. RELATIVE ERROR DYNAMIC MODEL EQUATION

As the leader satellite moves in orbit (figure 1), a certain desired location for the follower satellite also moves with some offset from the leader position. Let the position of the leader satellite be $\vec{r}_l$, the desired position of the follower be $\vec{r}_{des}$, and the follower satellite position be $\vec{r}_f$. The position of the desired position relative to the follower position is:

$$\vec{r}_{df} = \vec{r}_{des} - \vec{r}_f \tag{1}$$

This may be called the error in follower relative position. The desired follower position relative to the leader position is:

$$\vec{r}_{dl} = \vec{r}_{des} - \vec{r}_l \tag{2}$$

The acceleration of the error in follower relative position can then be written as:

$$\ddot{\vec{r}}_{df} = \ddot{\vec{r}}_{des} - \ddot{\vec{r}}_f = \ddot{\vec{r}}_l + \ddot{\vec{r}}_{dl} - \ddot{\vec{r}}_f \tag{3}$$

Recall from Kepler dynamics for two body motion:

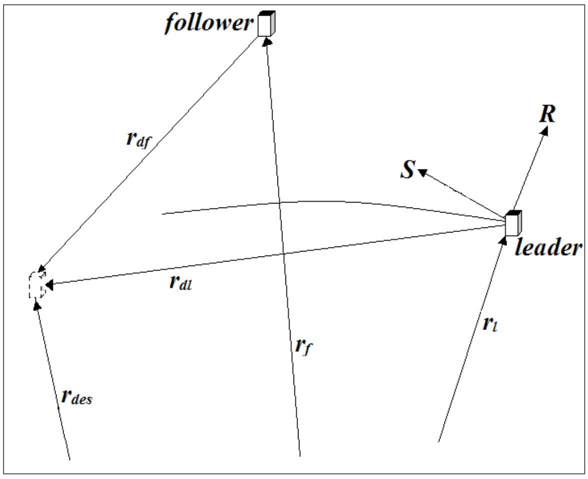
$$\ddot{\vec{r}}_l = -\mu \frac{\vec{r}_l}{r_l^3} \tag{4}$$

$$\ddot{\vec{r}}_f = -\mu \frac{\vec{r}_f}{r_f^3} + \vec{u} \tag{5}$$

where $\vec{u}$ is the control thrust vector. By assuming that $\ddot{\vec{r}}_{dl} = 0$, this is forced by control objective.

$$\therefore \ddot{\vec{r}}_{df} = \mu \frac{\vec{r}_f}{r_f^3} - \vec{u} - \mu \frac{\vec{r}_l}{r_l^3} \tag{6}$$

Let x, y, and z be the components of the vector $\ddot{\vec{r}}_{df}$ expressed in the RSW coordinate frame as shown in figure 1. The center of the RSW frame is located at the leader satellite center, where R is a unit vector pointing in direction from Earth center to satellite center, S is a unit vector in the velocity Figure 1 Relative positions of satellite in an orbit direction normal to R, and W completes the orthonormal set. Then the above dynamic model can be linearized in a similar way to that mentioned in [12, 13] to yield:



$$\ddot{x} = -f_x + 2\omega\dot{y} + \dot{\omega}y + \left(\omega^2 - \frac{2\mu}{r_{tgt}^3}\right)x$$

$$\ddot{y} = -f_y - 2\omega\dot{x} - \dot{\omega}x + \left(\omega^2 + \frac{\mu}{r_{tgt}^3}\right)y \tag{7}$$

$$\ddot{z} = -f_z + \left(\frac{\mu}{r_{tgt}^3}\right)z$$

According to the model given above in equation 7, the z dynamics are decoupled from the orbital plane dynamics and so can be controlled separately. In this deployment only the orbital plane dynamics controlled.

III. MODEL PREDICTIVE CONTROL ALGORITHM

The core of the MPC algorithm is the model of the plant, which can be in the form of a discrete state as follows:

$$\begin{aligned} x(k+1) &= f(x(k), \Delta u(k)) \\ y(k) &= h(x(k), \Delta u(k)) \end{aligned} \tag{8}$$

With this model form, the future output response of the plant can be predicted p-step ahead into the future $\hat{y}(k+l)$, where $l = 1, \dots\dots, p$. The prediction value $\hat{y}(k+l)$ depends on the past actuation and the planned *m-step* ahead actuation $\{\Delta u(k+j), j = 1, \dots, m-1, m < p\}$. The planned moves $\{\Delta u(k+j), j = 1, \dots\dots, m-1\}$ are determined as a solution to the following optimization problem.

$$J = \gamma^y \sum_{i=1}^{p} (e(k+i/k))^2 + \gamma^u \sum_{i=0}^{m-1} (\Delta u(k+i/k))^2 \tag{9}$$

Where, it can be noticed that the cost function index J incorporates the errors $e(k+i/k)$ which is the difference between the future reference trajectory $r(k+i/k)$ and the predicted output of the system $\hat{y}(k+i/k)$ equation 10, the change in the actuation moves $\Delta u(k+i/k)$, and the weighting output γ^y and input γ^u.

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$$e(k+i/k) = r(k+i/k) - \hat{y}(k+i/k) \quad subject\ to$$
$$x(k+i/k) = f(x(k+i/k), \Delta u(k+i/k) \quad for \quad i \geq 0 \tag{10}$$

Outside the control horizon m, the actuation moves are constant and their change *Δu(k+i/k) = 0*. The first element of the minimizing control sequence is implemented on the actual plant. Then the whole cycle of output measurement, prediction, and input trajectory determination is repeated. This procedure is repeated one sampling interval later with a new prediction horizon, control horizon and reference trajectory defined and new output measurement. Because the prediction horizon remains of the same length as for the previous sampling interval, but slides along by one sampling interval at each step, this way of control is called receding horizon strategy; the receding horizon strategy makes a closed loop control law from the original open loop using the actual state and output measurement of the plant under control.

The optimal control sequence depends on the current measurement *y(k/k)*, the prediction horizon *p*, the control horizon *m*, and the weights γ^y and γ^u. One of the advantages of the MPC algorithm is its applicability to handle in straightforward way multivariable interactive control problems, and to extend to constrained control problems.

IV. LINEAR CONTROL BASED ON LYAPUNOV FUNCTION

For the time variant system (LTV) in equation 7, assume a Lyapunov function [6] of the form:

$$V = \frac{1}{2}\left(k_x x^2 + k_y y^2\right) + \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right)$$
$$\therefore \frac{dV}{dt} = \left(k_x x\dot{x} + k_y y\dot{y}\right) \tag{11}$$

Substituting for acceleration $\ddot{x}, \ddot{y}$ from equation 7 yields,

$$\frac{dV}{dt} = \left(k_x x\dot{x} + k_y y\dot{y}\right) - \dot{x}f_x - \dot{y}f_y + \dot{\omega}(\dot{x}y - x\dot{y})$$
$$+\omega^2(x\dot{x} + y\dot{y}) - \frac{\mu}{r_{tgt}^3}(2x\dot{x} - y\dot{y}) \tag{12}$$

Let the control be as follow:

$$f_x = \left(k_x + \omega^2 - \frac{2\mu}{r_{tgt}^3}\right)x + k_{dx}\dot{x} + \dot{\omega}y + 2\omega\dot{y}$$
$$f_y = \left(k_y + \omega^2 + \frac{\mu}{r_{tgt}^3}\right)y + k_{dy}\dot{y} - \dot{\omega}x - 2\omega\dot{x} \tag{13}$$
$$\therefore \frac{dV}{dt} = -k_{dx}\dot{x}^2 - k_{dy}\dot{y}^2$$

Which is negative semi-definite, the equilibrium state can be easily checked by setting:

$$\frac{dV}{dx} = \frac{dV}{d\dot{x}} = \frac{dV}{dy} = \frac{dV}{d\dot{y}} = 0.$$

This makes

$$x = 0, y = 0, \dot{x} = 0, \dot{y} = 0$$

By applying the controls to the equation of motion 7, the closed loop system is:

$$\ddot{x} = -k_{dx}\dot{x} - k_x x$$
$$\ddot{y} = -k_{dy}\dot{y} - k_y y \tag{14}$$

V. SIMULATION RESULTS

A simulation tool was developed based on the MPC Toolbox in the MatLab/Simulink environment. First, an open response of the system to error initial conditions was obtained using the linear model of the system based on controller gains corresponding to $\omega_{nx} = \omega_{ny} = 0.0005$ *rad/second*, and $\xi_x = \xi_y = 0.65$. I t can be noticed from figure 2 that the time for the positions and acceleration of the system to return back to their zero initial conditions is long enough.

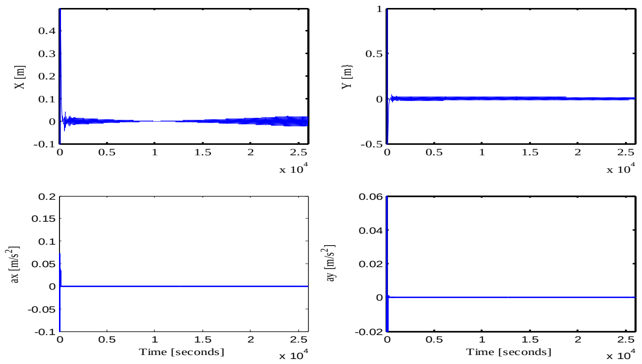

Fig 2: linear system response for errors initial conditions

The objective of this work is to test the ability and cabaplity of the linear controllers to bring bring the follower to be in distance of 100 m in the *x-dirextion* and 150 in the *y-direction* from the leader. An elliptical orbit for the leader was selected with the following parameters [6]: semi-major axis is 6.7781e+006 m, eccentricity 0.005 and inclination of 96°, the follower positon is given as the initial conditions for the *x* and *y* positions.

The closed loop response of the system having parameters similar to those implementd in figure 2 hows unstability to bring the system to the desired values. Hence the controller gains were modified to be $\omega_{nx} = \omega_{ny} = 0.003$ *rad/second*, and $\xi_x = \xi_y = 0.65$. These parameters give a good and fast closed loop response of the system as shown in the following figure 3.

The model predictive control (MPC) algorithm is applied to the system in the interest to get improved trajectroy tracking. For the purpose of testing the MPC algorithm, the same formation configuration as in the previous controller case is used. The closed loop system response based on the MPC algorithm is shown in figure 4. It can be noticed for figure 4 that the MPC drove the system to the desired *x* and *y* positions in short time comparing to the controller based on Lyapunov function and maintain zero error in the control interval.

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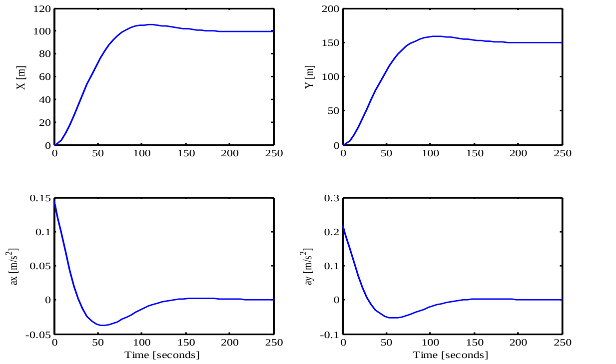

Fig 3: closed loop system response based on Lyapunov function

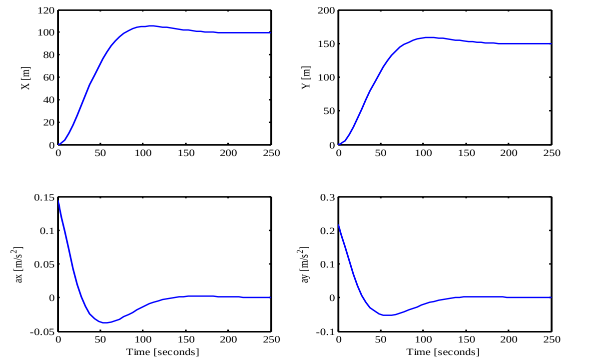

Fig 4: closed loop system response based on MPC algorithm

VI. CONCLUSION

This work demonstrated the feasibility of maintaining the formation conditions in an eccentric orbit in a prescribed interval. Two controllers are evaluated; a linear Lyapunov function type controller and model predictive control algorithm via simulation in the MatLab/Simulink environment. Both controllers sow ability to control the formation and correct the initial errors. MPC take less time to reach the desired positions comparing to the controller based Lyapunov function, Moreover, the control gains for the controller based Lyapunov function needs to be tuned by simulation to meet the prescribed behavior, and some gains may lead to instability.

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