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Free oscillations of two concentrically located cylindrical shells with a fluid between them

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Abstract- ***Free vibrations of two concentrically located cylindrical shells with liquid between them are studied. Such systems are met with in heat exchangers. The equations of motion independent on axial direction are written on the technical theory of cylindrical shells. The fluids motion is potential and is written by a wave equation. The fluid moves not separated from the cylinder walls. The fluid pressure in taken into account in the equations of motion of shells, and velocities of the fluid and the shell are equated on the boundaries. Representing the solution in the harmonic form, it is reduced to the system of transcendental equations. By comparing the solution of the problem without liquid and the solution in availability of liquid, the dependence of the frequency of the liquid less system with the frequency of the system with liquid is found. Eigen frequencies of vibrations are determined at some values of the system parameters, influence of the cylinders size on free vibrations of the cylinder is also studied.***

Index Terms: **density, frequency, pressure, shell, vibrations, and wave.**

I. INTRODUCTION

For the calculation of Eigen frequencies and amplitudes of the oscillations of the elastic element in the liquid, for instance, of the cylindrical heat generating element in the atomic reactor or of the tube in the heat exchange device, it is necessary to know the measure of the mass connected to it and to the damping force. As it is demonstrated in the experiments [1]-[6] the viscosity of the liquid has a great effect to the value of the connected mass and to the damping of vibration. More over, these properties depend on the location of the non-moveable borders surrounding the cylinder. In [7] the cross- section vibrations of the infinite cylinder in the liquid surrounding by the non-moving concentric shell are investigated. At this case the velocity of the cylinder is changed by the harmonic law and the small displacements are investigated when the amplitude of vibrations is significantly less than the dimension of the gap between the cylinders and the liquid flow originated by this effect is a durable one. It is shown that there is a big dependence between the hydro-dynamical damping of vibrations and the liquid viscosity and the frequencies of vibrations. In [8] iinvestigation of free vibrations of spherical inclusion containing elastically suspended mass situated in acoustic medium, by the inverse method.

II. THE PROBLEM FORMULATION.

In this work the free oscillations of two concentrically located shells are examined for the case when there is a fluid between the shells. The considered shell system is in great interest area of heat exchange researchers. On the base of the technical theory the equation of the movement of shells for the case when no displacement occurs in the axially direction can be presented in the form [9]:

$$\frac{1}{R_1^8}\frac{\partial^8\phi_1}{\partial\beta^8}+\frac{12\left(1-\nu^2\right)}{h_1^2R_1^6}\frac{\partial^4\phi_1}{\partial\beta^4}+\frac{q_1}{Eh_1R_1^4}\frac{\partial^6\phi_1}{\partial\beta^4\partial t^2}+z_1=0$$

$$\frac{1}{R_2^8}\frac{\partial^8\phi_2}{\partial\beta^8}+\frac{12\left(1-\nu^2\right)}{h_2^2R_2^6}\frac{\partial^4\phi_2}{\partial\beta^4}+\frac{q_2}{Eh_2R_2^4}\frac{\partial^6\phi_2}{\partial\beta^4\partial t^2}+z_2=0$$

(1)

where ϕ is displacement potential, β - the coordinate in the circumferential direction, R -radius of the shell, h - thickness, E - Young modulus, $z-$ fluid pressure, q - specific mass of the shell. In the case of the potential movement of the liquid for small displacements

$$z_1=-\rho\left.\frac{\partial\varphi}{\partial t}\right|_{r=r_1}, \qquad z_2=-\rho\left.\frac{\partial\varphi}{\partial t}\right|_{r=r_2}$$

(2)

ρ -fluid density and φ - fluid potential.

The potential of the fluid velocity satisfies the equation:

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial r^2}=\frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}$$

(3)

By connecting the potential of the liquid velocity with the velocity of the shell displacements on the borders we have:

$$\frac{\partial^5\phi_1}{\partial\beta^4\partial t}=\left.\frac{\partial\varphi}{\partial r}\right|_{r=r_1};\quad \frac{\partial^5\phi_2}{\partial\beta^4\partial t}=\left.\frac{\partial\varphi}{\partial r}\right|_{r=r_2}$$

(4)

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For the purpose of investigation of free oscillations the particular solutions of the system (1), (4) are considered:

$$\phi_i = \phi_i^0 e^{i\omega t}\sin n\beta \qquad \text{и} \qquad \varphi = \varphi_n^0 e^{i\omega t}\sin n\beta$$
(5)

By substituting (5) into (1) and (4) we get

$$\frac{n^8}{R_1^8}\phi_1^0 + \frac{12(1-\nu)n^4}{h_1^2R_1^6}\phi_1^0 - \frac{n^2q_1\omega^2}{Eh_1R_1^4}\phi_1^0 + \frac{i\omega\rho}{Eh_1}\varphi_n^0\Big|_{r=r_1} = 0$$
$$\frac{in^4\omega}{R_1^4}\phi_1^0 = \frac{\partial\varphi_n^0}{\partial r}\Big|_{r=r_1}$$

(6)

$$\frac{n^8}{R_2^8}\phi_2^0 + \frac{12(1-\nu)n^4}{h_2^2R_2^6}\phi_2^0 - \frac{n^2q_2\omega^2}{Eh_2R_2^4}\phi_2^0 + \frac{i\omega\rho}{Eh_1}\varphi_n^0\Big|_{r=r_2} = 0$$
$$\frac{in^4\omega}{R_2^4}\phi_2^0 = \frac{\partial\varphi_n^0}{\partial r}\Big|_{r=r_2}$$

By excluding ϕ_1^0 and ϕ_2^0 from (6) we get

$$\frac{n^4}{R_1^4}\frac{\partial\varphi_n}{\partial r_1} + \frac{12(1-\nu^2)}{h_1^2R_1^2}\frac{\partial\varphi_n}{\partial r_1} - \frac{q_1\omega^2}{Eh_1n^2}\frac{\partial\varphi_n}{\partial r_1} - \frac{\omega\rho}{Eh_1}\varphi_n\Big|_{r=r_1} = 0$$
$$\frac{n^4}{R_2^4}\frac{\partial\varphi_n}{\partial r_2} + \frac{12(1-\nu^2)}{h_2^2R_2^2}\frac{\partial\varphi_n}{\partial r_2} - \frac{q_2\omega^2}{Eh_2n^2}\frac{\partial\varphi_n}{\partial r_1} - \frac{\omega\rho}{Eh_1}\varphi_n\Big|_{r=r_2} = 0$$

(7)

In the case when there is no liquid between the shells, i.e. when $(\rho = 0)$ the system (7) takes the form:

$$\frac{n^4}{R_1^4} + \frac{12(1-\nu^2)}{h_1^2R_1^2} - \frac{q_1\omega_1{}^2}{Eh_1n^2} = 0$$
$$\frac{n^4}{R_2^4} + \frac{12(1-\nu^2)}{h_2^2R_2^2} - \frac{q_2\omega_2^2}{Eh_2n^2} = 0$$

(8)

where ω_1 and ω_2 are the frequencies of free oscillations of the cylinders without any surrounding medium. By combining the equations (7) and (8) we get

$$\frac{q_1(\omega_1^2-\omega^2)}{n^2}\frac{\partial\varphi_n}{\partial r_1} - \omega^2\rho\varphi_n\big|_{r=r_1} = 0$$
$$\frac{q_2(\omega_2^2-\omega^2)}{n^2}\frac{\partial\varphi_n}{\partial r_2} \pm \omega^2\rho\varphi_n\big|_{r=r_2} = 0$$
(9)

or

$$\frac{q_1(\omega_1^2-\omega^2)}{n^2}\varphi_{n1}'\Big|_{r=r_1} - \omega\rho a\varphi_n\big|_{r=r_1} = 0$$
$$\frac{q_2(\omega_2^2-\omega^2)}{n^2}\varphi_{n2}'\Big|_{r=r_2} \pm \omega\rho a\varphi_n\big|_{r=r_2} = 0$$
(10)

By substituting the solutions of the equation (3) in (10) $\varphi_n = B_1J_n + B_2N_n$ (where J_n and N_n are the Bessel's and respectively Neumann's functions) into (10) we have

$$q_1(\omega_1^2-\omega^2)\left[B_1J_n'\left(\frac{\omega r_1}{a}\right) + B_2N_n'\left(\frac{\omega r_1}{a}\right)\right] - \rho\omega a\left[B_1J_n\left(\frac{\omega r_1}{a}\right) + B_2N_n\left(\frac{\omega r_1}{a}\right)\right] = 0$$
$$q_2(\omega_2^2-\omega^2)\left[B_1J_n'\left(\frac{\omega r_2}{a}\right) + B_2N_n'\left(\frac{\omega r_2}{a}\right)\right] \pm \rho\omega a\left[B_1J_n\left(\frac{\omega r_2}{a}\right) + B_2N_n\left(\frac{\omega r_2}{a}\right)\right] = 0$$
(11)

Taking into account that B_1 and B_2 unknown in (11) we have

$$\left[q_1(\omega_1^2-\omega^2)J_n'\left(\frac{\omega r_1}{a}\right) - \rho\omega aJ_n\left(\frac{\omega r_1}{a}\right)\right]B_1 + \left[q_1(\omega_1^2-\omega^2)N_n'\left(\frac{\omega r_1}{a}\right) - \rho\omega aN_n\left(\frac{\omega r_1}{a}\right)\right]B_2 = 0$$
$$\left[q_2(\omega_2^2-\omega^2)J_n'\left(\frac{\omega r_2}{a}\right) \pm \rho\omega aJ_n\left(\frac{\omega r_2}{a}\right)\right]B_1 + \left[q_2(\omega_2^2-\omega^2)N_n'\left(\frac{\omega r_2}{a}\right) \pm \rho\omega aN_n\left(\frac{\omega r_2}{a}\right)\right]B_2 = 0$$
(12)

For the case $\omega_1 = \omega_2 = \omega_0$ (what we can obtain by taking up the parameters R, h, q in (8) by the corresponding way) we have

$$\left[q_1J_n'\left(\frac{\omega r_1}{a}\right) - \frac{\rho\omega a}{\omega_0^2-\omega^2}J_n\left(\frac{\omega r_1}{a}\right)\right]B_1 + \left[q_1N_n'\left(\frac{\omega r_1}{a}\right) - \frac{\rho\omega a}{\omega_0^2-\omega^2}N_n\left(\frac{\omega r_1}{a}\right)\right]B_2 = 0$$
$$\left[q_2J_n'\left(\frac{\omega r_2}{a}\right) - \frac{\rho\omega a}{\omega_0^2-\omega^2}J_n\left(\frac{\omega r_2}{a}\right)\right]B_1 + \left[q_2N_n'\left(\frac{\omega r_2}{a}\right) \pm \frac{\rho\omega a}{\omega_0^2-\omega^2}N_n\left(\frac{\omega r_2}{a}\right)\right]B_2 = 0$$
(13)

By denoting $\Omega = \dfrac{\rho \omega a}{\omega_0^2 - \omega^2}$ and taking into account the condition of non-triviality of the solution of the system (13) we get:

$$\left[q_1 J'\left(\frac{\omega r_1}{a}\right) - \Omega J\left(\frac{\omega r_1}{a}\right)\right]\left[q_2 N_n'\left(\frac{\omega r_2}{a}\right) \pm \Omega N_n\left(\frac{\omega r_2}{a}\right)\right] -$$
$$-\left[q_2 J'\left(\frac{\omega r_2}{a}\right) \pm \Omega J_n\left(\frac{\omega r_2}{a}\right)\right]\left[q_1 N_n'\left(\frac{\omega r_1}{a}\right) - \Omega N_n\left(\frac{\omega r_1}{a}\right)\right] = 0$$

or

$$\left[J_n\left(\frac{\omega r_1}{a}\right) N'\left(\frac{\omega r_2}{a}\right) \pm J_n\left(\frac{\omega r_2}{a}\right) N_n\left(\frac{\omega r_1}{a}\right)\right]\Omega^2 \pm \left[J_n'\left(\frac{\omega r_1}{a}\right) N\left(\frac{\omega r_2}{a}\right) \pm J_n\left(\frac{\omega r_1}{a}\right) N'\left(\frac{\omega r_2}{a}\right) -\right.$$
$$\left. - J_n'\left(\frac{\omega r_2}{a}\right) N_n\left(\frac{\omega r_1}{a}\right) \pm J_n\left(\frac{\omega r_2}{a}\right) N_n'\left(\frac{\omega r_1}{a}\right)\right]\Omega + J_n'\left(\frac{\omega r_1}{a}\right) N_n'\left(\frac{\omega r_2}{a}\right) - J_n'\left(\frac{\omega r_2}{a}\right) N_n'\left(\frac{\omega r_1}{a}\right) = 0$$

(14)

By solving (14) in the relation to Ω we get

$$\Omega = \frac{\beta \pm \sqrt{\beta^2 - 4\alpha c}}{2\alpha}$$

where

$$\alpha = J_n\left(\frac{\omega r_1}{a}\right) N_n\left(\frac{\omega r_2}{a}\right) \pm J_n\left(\frac{\omega r_2}{a}\right) N_n\left(\frac{\omega r_1}{a}\right)$$

$$\beta = J_n'\left(\frac{\omega r_1}{a}\right) N_n\left(\frac{\omega r_2}{a}\right) + J_n\left(\frac{\omega r_1}{a}\right) N_n'\left(\frac{\omega r_2}{a}\right) - J_n'\left(\frac{\omega r_2}{a}\right) N_n\left(\frac{\omega r_1}{a}\right) \pm J_n\left(\frac{\omega r_2}{a}\right) N_n'\left(\frac{\omega r_2}{a}\right)$$

$$c = J_n'\left(\frac{\omega r_1}{a}\right) N'\left(\frac{\omega r_2}{a}\right) - J_n'\left(\frac{\omega r_2}{a}\right) N_n'\left(\frac{\omega r_1}{a}\right)$$

For some given values of the parameters of the problem the graphs $K(\omega) - \omega$ are constructed at the intervals $0 - 3 \cdot 10^6$ for $K(\omega)$ and $0 - 4{,}5 \cdot 10^4$ for ω (fig 1 $(q = 50,\ r_2 = 0{,}3,\ \ r_1 = 0{,}1)$), the graphs $K(\omega) - \omega$ are constructed at the interval $0 - 2{,}5 \cdot 10^6$ for $K(\omega)$ and $0 - 4{,}5 \cdot 10^4$ for ω (fig 2 $(q = 30,\ r_2 = 0{,}3,\ \ r_1 = 0{,}1)$), the graphs $K(\omega) - \omega$ are constructed at the interval $0 - 5 \cdot 10^6$ for $K(\omega)$ and $0 - 4 \cdot 10^4$ for ω (fig 3 $(q = 50,\ r_2 = 0{,}5,\ \ r_1 = 0{,}3)$).

III. CONCLUSIONS

The value of the Eigen frequency of the system increases quickly by the increasing of the frequency of the free cylinders. Then the increasing of the frequency of the system goes much slower and approximates asymptotically to the certain value. The frequency of the second mode beginning increases from some value with the decreasing of the stiffness of the tubes up to the certain value. This can be explained by the movement of tube walls and fluid particles that occurs at the same. First found by analytical solution of the free vibrations of concentric tubes. Such a tube system can be used in heat exchangers, heating or cooling devices. Meaning Eigen frequencies tubes may be necessary to prevent the resonance phenomena in the vicinity of sources of vibration or, conversely, cause fluctuations in the system to enhance heat transfer.

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APPENDIX

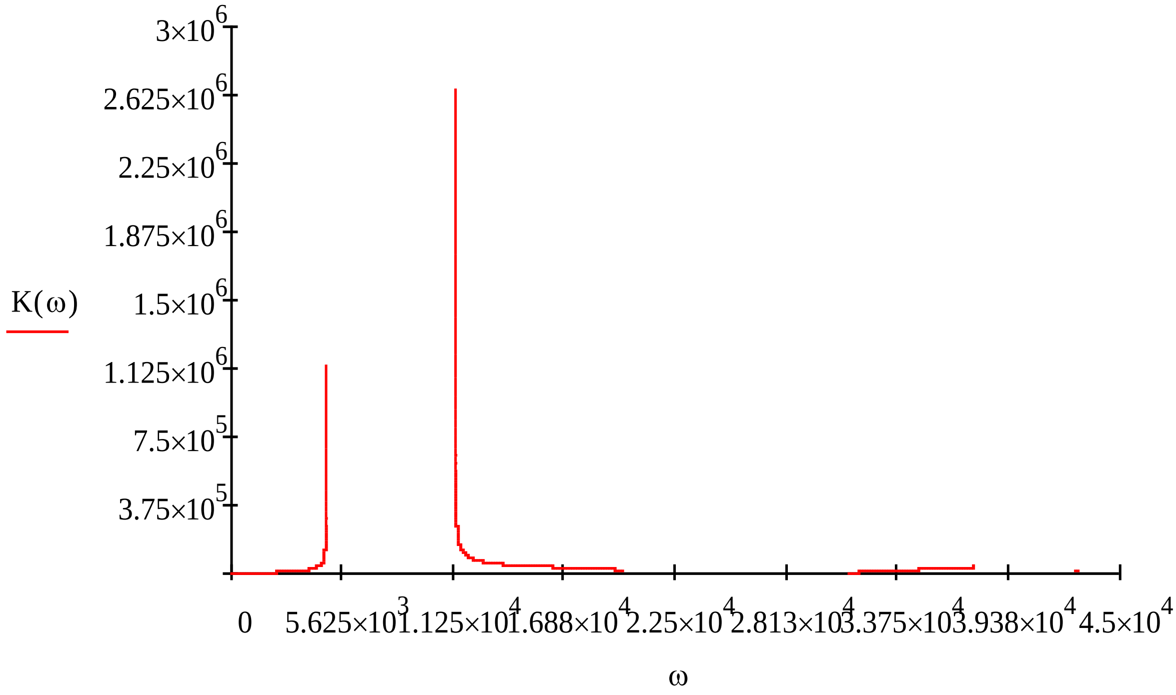

Fig 1: The dependences of the frequencies of oscillations for different modes of the system on the frequency of the empty shell

$$\left(q = 50,\ r_2 = 0{,}3,\ \ r_1 = 0{,}1\right)$$

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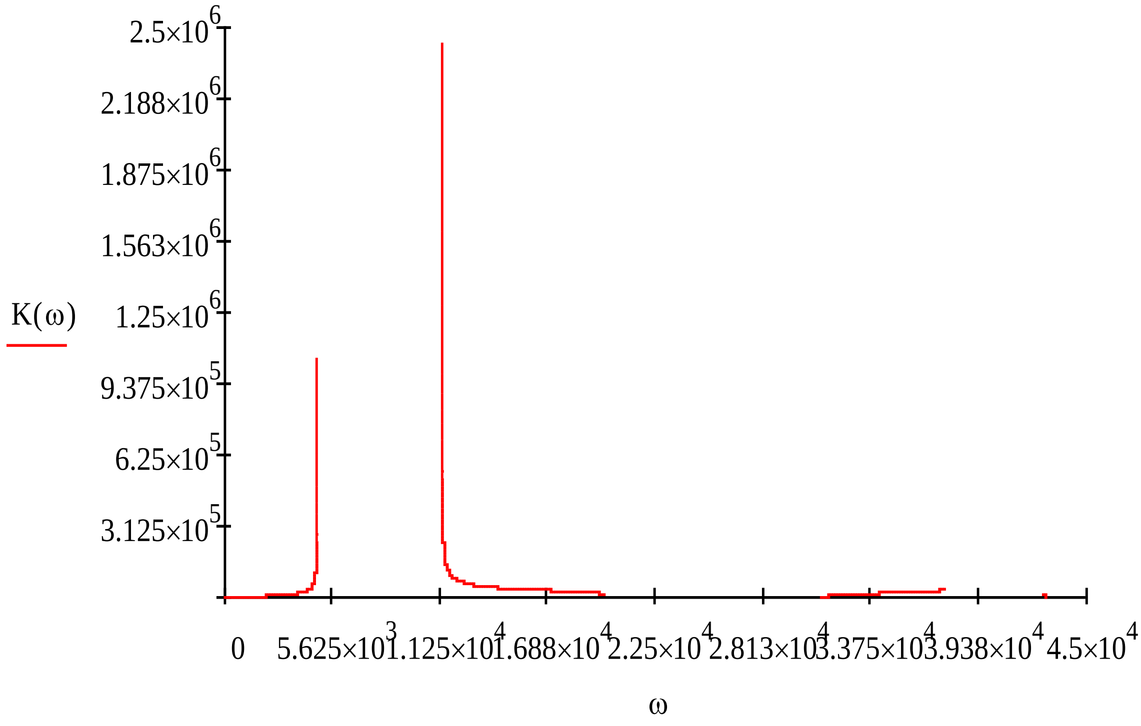

Fig 2: The dependences of the frequencies of oscillations for different modes of the system on the frequency of the empty shell

$$(q = 50,\ r_2 = 0{,}3,\ \ r_1 = 0{,}1)$$

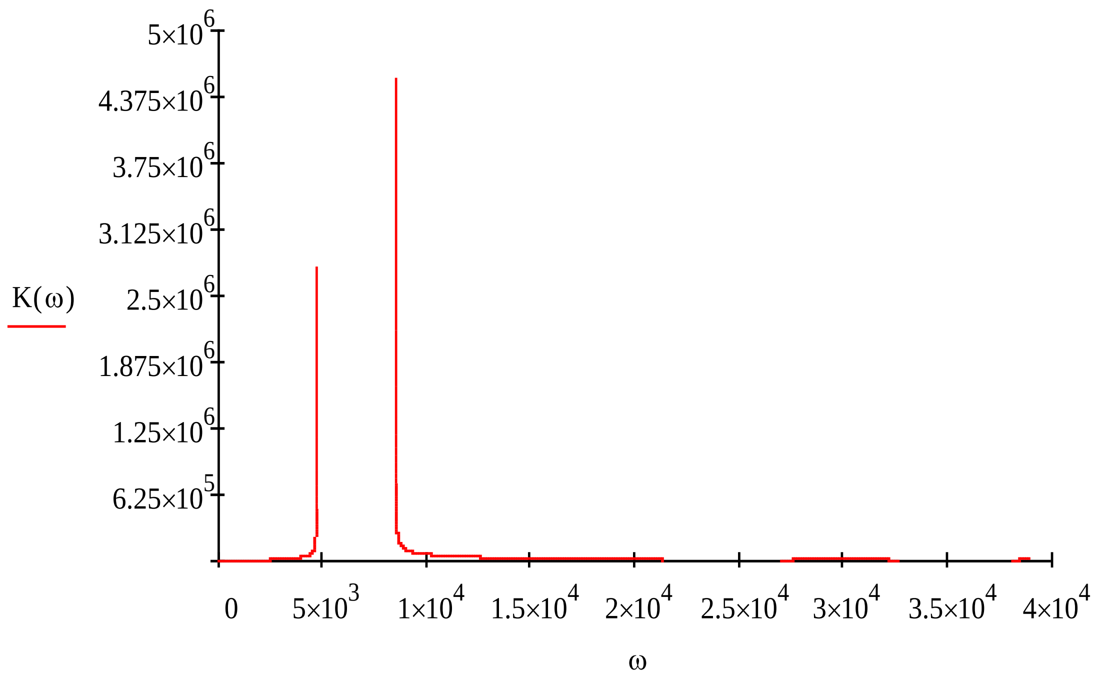

Fig 3: The dependences of the frequencies of oscillations for different modes of the system on the frequency of the empty shell

$$(q = 50,\ r_2 = 0{,}5,\ \ r_1 = 0{,}3)$$