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# Optimal Control of Wastewater Biological Treatment

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***Abstract- The aim of this study is concerned with control problem of the biological treatment of wastewater using a fixed bed bioreactor. The dynamics of such process is described by a distributed parameter system. Microorganisms fixed in the bioreactor absorb the substrate in such a way that the substrate concentration decreases in the out flowing water. The process must regulate the substrate concentration at the outlet of the bioreactor between two prescribed levels of concentration. This problem is solved via constrained controllability techniques that lead to algorithm successfully tested by simulations.***

***Keywords:*** **Constrained-controllability, Distributed system, Uzawa algorithm, Wastewater treatment.**

## I. INTRODUCTION

Human impacts on ecosystems have reached global proportions and have disrupted a wide range of valuable ecological services, thus, it represents threats to the humanity health. Hence the safeguarding of the ecosystems and natural environment becomes a major economic, technological and environmental challenge. Among the various methods applied, the biological treatment processes are widely used, particularly in the wastewater epuration treatment. Such process uses existing fauna and flora in the water used by facilitating the assimilation of the organic matter, nitrogen and phosphorus by oxygen contribution. The goal is to reduce or remove organic matter, solids, nutrients and other pollutants from wastewater before the remaining water, called effluent, is discharged back to the environment. Fixed bed bioreactor is a biological treatment process that uses a support (usually rock, sand or plastic) contained in a tank that serves as a surface on which microorganisms growth occurs. Wastewater spreads over the support, the microorganisms absorb the pollutants as their food supply and degrade organic compounds fast and reliably. As the organic matter and nutrients are absorbed, the film of microorganisms grows and thickens. Trickling filters, rotating biological contactors, and sand filters are examples of fixed bed reactors. Choosing the right bioreactor system for a particular application is governed by several factors, the influent flow rate, the mix and concentration of organic contaminants, the buildup and retention of biomass and the required effluent quality. As for all industrial process, a continuous regulation is necessary in order to optimize the operations and to maintain the degradation of the substrate polluting on a wished level at the exit of the treatment unit.

In the considered problem, we try to regulate the substrate concentration at a bottom sub region of the reactor [1] between two prescribed levels by using a fixed bed bioreactor and to give an application of parabolic system constrained-controllability [2], [3], connected to the above problem. This is the aim of this paper which is presented as follows

The next section is devoted to give definition and some properties of the regional constrained-controllability. In the third section, we present useful results from an implementation point of view for the computation of the optimal control. The last section is focused on the model of the fixed bed bioreactor process and numerical illustration results are given.

## II. EXTENDED CONTROL PROBLEM

Let Ω be a regular bounded set of $IR^n$ $(n \geq 1)$ with boundary.

For a given time T> 0, let Q = $\Omega \times ]0,T[$ and $\Sigma = \partial\Omega \times ]0,T[$.

We consider a parabolic system excited by controls which may be applied via various types of actuators given by the following equation :

$$\begin{cases} \dfrac{\partial y}{\partial t}(x,t) = Ay(x,t) + Bu(t) & \text{in } Q \\ y(x,0) = y^0(x) & \text{in } \Omega \\ \dfrac{\partial y}{\partial \nu_A}(\xi,t) = 0 & \text{in } \Sigma \end{cases} \quad (1)$$

Where $A$ is elliptic second-order linear differential operator with smooth coefficients that do not depend on t and generates a strongly continuous semi-group $(S(t))_{t\geq 0}$ on the Hilbert state space $L^2(\Omega)$, $\dfrac{\partial y(\xi,t)}{\partial \nu_A}$ indicates the conormal with respect to

$A.\ B \in L(IR^m, L^2(\Omega)), y^0 \in L^2(\Omega)$ and

$u \in U := L^2(0,T,IR^m)$ , the space controls endowed with the scalar product <.,.> and the norm $\|.\|$ .

Denote by $y_u(.)$ the solution of (1) when it is excited by a control $u$ , suppose that $y_u(T) \in H^1(\Omega)$ and consider

• $\Gamma$ a nonempty sub region of $\partial\Omega$ which may be connected or not and assumed to be of Lebesgue positive measure.

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• $\gamma_0 : H^1(\Omega) \to H^{\frac{1}{2}}(\partial\Omega)$ the trace operator of order zero which is linear, continuous and Surjective;

• the operator restriction

$$\chi_\Gamma : H^{\frac{1}{2}}(\partial\Omega) \to H^{\frac{1}{2}}(\Gamma)$$
$$z \to \chi_\Gamma z = z|_\Gamma$$

while $\chi_\Gamma^*$ is considered for the adjoint operator.

Now let $H : U \to H^1(\Omega)$ defined by

$$\forall u \in U,\ Hu = \int_0^T S(T-s)Bu(s)ds.\ ,\ \alpha(.) \text{ and } \beta(.)$$

be two given real functions in $H^{\frac{1}{2}}(\Gamma)$ such that $\alpha(.) \le \beta(.)$ on $\Gamma$, and set

$$[\alpha(.), \beta(.)] = \{ y \in H^{\frac{1}{2}}(\Gamma) \mid \alpha(.) \le y(.) \le \beta(.) \}$$

on $\Gamma$ .

***Definition 2.1***

We say that the system (1) is $[\alpha(.), \beta(.)]-$ controllable on $\Gamma$ if there exists $u \in U$ such that $\alpha(.) \le \chi_\Gamma(\gamma_0 y_u(T)) \le \beta(.)$ on $\Gamma$.

It is clear that the system (1) is $[\alpha(.), \beta(.)]-$controllable on $\Gamma$ if $[\alpha(.), \beta(.)] \cap Im\chi_\Gamma\gamma_0 H \neq \varnothing$.

***Remarque* 2.2**

1. The above definition means that we are only interested in the transfer of the system (1) to a state between $\alpha(.)$ and $\beta(.)$ on $\Gamma$.
2. The control u depends on the time variable but it also implicitly depends on $\Gamma$.
3. The above definition doesn't allow for point wise or boundary controls as for such systems $B \notin L(IR^m, L^2(\Omega))$ and the solution $y_u(T) \notin L^2(\Omega)$. However, the extension can be carried out in a similar manner if one takes regular controls such that $y_u(T) \in H^1(\Omega)$.

We are interested by the following minimization problem

$$\begin{cases} \inf \frac{1}{2} \| u \|^2 \\ u \in U_{ad} \end{cases} \qquad (2)$$

where $U_{ad} = \{ u \in U \mid \chi_\Gamma \gamma_0 y_u(T) \in [\alpha(.), \beta(.)] \}$.

If system (1) is $[\alpha(.), \beta(.)]$ -controllable on $\Gamma$ then problem (2) has a unique solution. Indeed $U_{ad} \neq \varnothing$, the mapping $u \to \frac{1}{2}\| u \|^2$ is strictly convex, coercive proper and lower semi-continuous in $U$ which is reflexive, and the mapping $u \to \chi_\Gamma \gamma_0 y_u(T)$ is linear, so $U_{ad}$ is convex.

We only have to verify that $U_{ad}$ is a closed subset of $U$. For this, consider a sequence $(u_n)_n \in U_{ad}$ such that $u_n \to u$ strongly in $U$. Since $\chi_\Gamma \gamma_0 H$ is continuous then $\chi_\Gamma \gamma_0 Hu_n \to \chi_\Gamma \gamma_0 Hu$ strongly in $H^{\frac{1}{2}}(\Gamma)$, but $\chi_\Gamma \gamma_0 y_{u_n}(T) \in [\alpha(.), \beta(.)]$ which is closed then $\chi_\Gamma \gamma_0 y_u(T) \in [\alpha(.), \beta(.)]$, this means that $u \in U_{ad}$, so $U_{ad}$ is closed. Hence (2) has a unique solution $u^*$ [4].

***Proposition* 2.3**

If the system $(1)$ is $[\alpha_1(.), \beta_1(.)]-$controllable on $\Gamma$, with $\alpha_1 = \alpha + \delta$, $\beta_1 = \beta - \delta, \delta > 0$, then $u^*$ the solution of $(2)$ is given by

$$u^* = -(\chi_\Gamma \gamma_0 H)^* \lambda^* \qquad (3)$$

where $\lambda^* \in H^{\frac{1}{2}}(\Gamma)$ satisfies

$$\begin{cases} G_\Gamma \lambda^* + z^* = 0 \\ z^* = P_{[\alpha(.),\beta(.)]}(\rho\lambda^* + z^*) \end{cases} \qquad (4)$$

with $P_{[\alpha(.),\beta(.)]} : H^{\frac{1}{2}}(\Gamma) \to [\alpha(.), \beta(.)]$ denotes the projection operator $G_\Gamma = (\chi_\Gamma \gamma_0 H)(\chi_\Gamma \gamma_0 H)^*$ and $\rho > 0$.

**Proof.**

The proof is based on the equivalence between the problem $(2)$ and the saddle point problem

$$\begin{cases} \inf \frac{1}{2} \| u \|^2 \\ (u, z) \in W \end{cases} \qquad (5)$$

Where

$$W = \{ (u, z) \in U \times [\alpha(.), \beta(.)] \mid \chi_\Gamma \gamma_0 y_u(T) - z = 0 \}$$

To problem (5) we associate the Lagrangian defined for all $(u, z, \lambda) \in U \times [\alpha(.), \beta(.)] \times H^{\frac{1}{2}}(\Gamma)$;

$$L(u, z, \lambda) = \frac{1}{2}\| u \|^2 + \langle \lambda, \chi_\Gamma \gamma_0 y_u(T) - z \rangle_{H^{\frac{1}{2}}(\Gamma)}$$

$L$ has a saddle point $(u^*, z^*, \lambda^*)$ and that $u^*$ is the required optimal control (see [5]).

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## III. NUMERICAL APPROACH

In this section we describe a numerical scheme which allows the computation of the optimal control. Consider the system (1) excited by one zone control $(D, g)$.

$$\begin{cases} \dfrac{\partial y}{\partial t}(x,t) = Ay(x,t) + \chi_D g(x)u(t) & \text{in } Q \\ y(x,0) = y^0(x) & \text{in } \Omega \\ \dfrac{\partial y}{\partial \nu_A}(\xi,t) = 0 & \text{in } \Sigma \end{cases} \quad (6)$$

Where $D \subset \Omega$ and $g \in L^2(D)$ is the spatial repartition of the control on $D$ and the solution is assumed to be in $L^2(0,T;H^1(\Omega))$. Assume that there exists a complete set of eigen functions $(\varphi_m)_{m\in I}$ of $A$ in $H^1(\Omega)$ associated with the eigen values $\gamma_m$.

For $x=(x_1,\cdots,x_n)\in\Omega$ and $m=(m_1,\cdots,m_n)\in I$, let $\bar{x}=(x_1,\cdots,x_{n-1})$ and $\bar{m}=(m_1,\cdots,m_{n-1})$. Suppose that the functions $\Psi_{\bar{m}}(\bar{x}) = \chi_\Gamma \gamma_0 \varphi_m(x), m\in I$, form a complete set in $H^{\frac{1}{2}}(\Gamma)$.

We have seen that the computation of the optimal control solution of the problem $(2)$ turns up to solve the equations $(3)$ and $(4)$ which may be done by the following algorithm which is of Uzawa type.

**Algorithm**:

1. Initial data: $\Gamma$, control zone, stopping criteria $\grave{o}$ small enough.

2. Initiate two functions $(z_0,\lambda_1)\in[\alpha(.),\beta(.)]\times H^{\frac{1}{2}}(\Gamma)$

3. $(z_{n-1},\lambda_n)$ known, we determine $u_n, z_n$ with the formulas

$$u_n(t) = -\sum_{m\in I} e^{\gamma_m(T-t)}\langle\varphi_m, g\rangle_D \langle\Psi_{\bar{m}},\lambda_n\rangle_{H^{\frac{1}{2}}(\Gamma)} \quad (7)$$

$$z_n(x) = \begin{cases} \alpha(x) & \text{if } \rho\lambda_n(x)+z_{n-1}(x) \le \alpha(x) \text{ a.e.} \\ z_{n-1}(x)+\rho\lambda_n(x) & \text{if } \alpha(x) \le z_{n-1}(x)+\rho\lambda_n(x) \le \beta(x) \text{ a.e.} \\ \beta(x) & \text{if } z_{n-1}(x)+\rho\lambda_n(x) \ge \beta(x) \text{ a.e.} \end{cases} \quad (8)$$

4. If $\| z_n - z_{n-1} \|_{H^{\frac{1}{2}}(\Gamma)} \le \grave{o}$, stop

else

$$\lambda_{n+1}(x) = \lambda_n(x) + \left(\sum_{m\in I}\Psi_{\bar{m}}(\bar{x})\langle\varphi_m,g\rangle_D \int_0^T e^{\gamma_m(T-t)}u_n(t)dt - z_n(x)\right) \quad (9)$$

and return to step 3.

If $(u^*,z^*,\lambda^*)$ is the saddle point of $L$, then the sequence $u_n$ converges to $u^*$ solution of the problem (2)and the sequence $z_n$ converges to $z^*$ [5].

## IV. WASTEWATER BIOLOGICAL TREATMENT PROBLEM

In this section we will establish a mathematical model that describes wastewater biological treatment and use the above algorithm for optimal regulation.

### *A. System description and model equations.*

Bioreactors have frequently been used to grow useful cells or to clean contaminated effluent, such as water. A bioreactor can be broadly defined as a container with a bio-substance therein. As a chemically active biological substance, it can be one of many nonliving's substances and living microorganisms. For example, the bio-substance may be a chemical such as an enzyme or hormone or a living entity such as a bacterial or viral species. The container of the bioreactor has taken various forms, including items as simple as a vat to more complex items like porous elements and microcapsules [6].

In a bioreactor with fixed-bed, the microorganisms are fixed on a stationary support as of polymeric or ceramic through which the substrate runs out in a continuous way. The biomass (the mass of these microorganisms) grows by elimination of the substrate and a product is appeared. Thus, the process can be represented by the fermentation equation $x+s\rightarrow p$, where $x$ is the biomass concentration (microorganisms), $s$ is the substrate concentration (food of the biomass) and $p$ is the product.

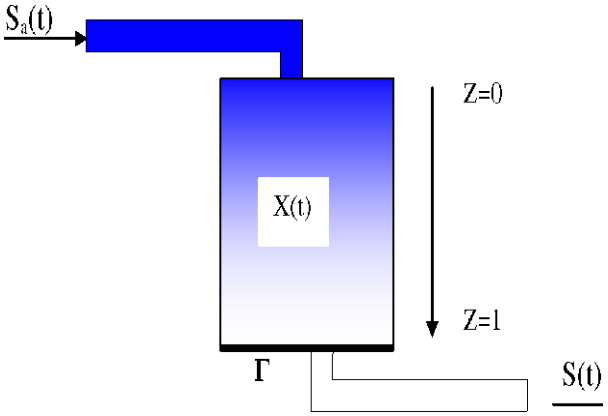


**Fig 1: Schematic view of the fixed-bed bioreactor.**

***Data***

- The control is the feed flow rate in substrate,
- The substrate concentration is known at the entry of the bioreactor,
- Measures of the substrate concentration at the outlet of the bioreactor, and at an internal point of the bioreactor.

We assume that the biomass and the substrate are uniformly distributed of the length of the bioreactor. The objective is to regulate the substrate concentration at the outlet of the bioreactor. Biomass variation is described by the equation:

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$$\frac{dVx}{dt} = \mu(.)Vx - DVx$$

where $\mu(.)Vx$ represents the biomass augmentation during $dt$ and $D$ the dilution rate. Whilst the substrate accumulation verify the equation :

$$\frac{dVs}{dt} = f(t)(s_a - s(z,t)) - k_1\mu(.)Vx$$

where $-k_1\mu(.)Vx$ is the quantity of the substrate consumed by the microorganisms during this augmentation. The model of the process can be carried out in one-dimensional space $(z \in \Omega = ]0,1])$ with a bioreactor whose culture volume $V$ is constant. Dividing by $V$ , the system is described by the following equations

$$\begin{cases} \frac{ds}{dt}(z,t) = u(t)(s_a - s(z,t)) - k_1\mu(.)x(z,t) \;\; ]0,1]\times[0,T] \\ \frac{dx}{dt}(z,t) = \mu(.)x(z,t) - Dx(z,t) \;\; ]0,1]\times[0,T] \\ s(z,0) = s_0(z),\ x(z,0) = x_0(z) \text{ on } ]0,1] \end{cases}$$

where $z$ , is the space variable, $s_a$ , the concentration of feed substrate, $k_1$ , yield coefficient, $\mu(.)$ , maximum specific growth rate of the microorganisms, $D$ , dilution rate, and $u(t) = \frac{f(t)}{V}$ , volumetric feed flow rate in the substrate and $V$ the cross sectional area of the bioreactor, where $f(t)$ indicates feed flow rate in substrate.

Among numerous models proposed in the literature, the Monod model is a classical one much used in microbiology to evaluate biodegradation processes and suggests that the microbial growth rate $\mu(.)$ and the substrate concentration $s$ are related by the equation:

$$\mu(s) = \mu_{max}\frac{s}{k_s + s}$$

where $\mu_{max}$ is the maximum specific growth rate, and $k_s$ the half saturation constant for growth for the limiting substrate. The Monod kinetic term $\frac{s}{k_s + s}$ represents the substrate limitation of a single substrate $s$ .

***B. Steady state model***

The steady state model is a situation in which all state variables are constant in spite of ongoing processes that strives to change them. For an entire system to be at steady state, there must be a flow through the system. Steady state points are the intersection of the nullclines of $s$ and $x$ :

$$\frac{ds}{dt} = 0 \quad \text{and} \quad \frac{dx}{dt} = 0.$$

Let $s^*$ and $x^*$ be the steady state points, and suppose that $u^*(t) = D = \text{ constant.}$

The equation $\frac{dx}{dt} = 0$ is equivalent to $(\mu^* - D)x^* = 0$

which gives $x^* = 0$ or $\mu^* = D$ , thus $s^* = \frac{Dk_s}{\mu_{max} - D}$.

and the equation $\frac{ds}{dt} = 0$ allows to write the equation

$$D(s_a - s^*(z)) - k_1\mu^* x^* = 0 \;\forall\; z \in ]0,1].$$

Two cases are possible

1. If $x^* = 0$ , then $D(s_a - s^*) = 0 \Rightarrow s^* = s_a$

and so $(0, s_a)$ is a trivial steady state. It corresponds to washout of the bioreactor (there is no microorganisms in the bioreactor).

(10)

2. If $s^* = \frac{Dk_s}{\mu_{max} - D}$, then $x^* = \frac{s_a - s^*}{k_1}$.

It follows that

$$\begin{cases} s^* = \frac{Dk_s}{\mu_{max} - D} \\ x^* = \frac{1}{k_1}(s_a - \frac{Dk_s}{\mu_{max} - D}) \end{cases} \qquad (11)$$

Consequently $(x^*, s^*)$ is a nontrivial steady-state point.

***C. Linearized model.***

The previous model (10) is nonlinear but the nature of the problem allows the study to be made in a linear context.To apply the previous result we will consider the linearized model of (10) around the steady state $(s^*, x^*, u^*)$.

Now let put

$$f_1(s,x,u) = u(t)(s_a - s(z,t)) - k_1\mu(s)x(z,t) \quad (12)$$

$$f_2(s,x) = (\mu(s) - D)x(z,t) \quad (13)$$

The linearized system is then

$$\begin{cases} \frac{\partial \delta s(z,t)}{\partial t} = \frac{\partial f_1(s,x,u)}{\partial s}_{(s^*,x^*,u^*)}\delta s(z,t) + \frac{\partial f_1(s,x,u)}{\partial x}_{(s^*,x^*,u^*)}\delta x(z,t) \\ \qquad + \frac{\partial f_1(s,x,u)}{\partial u}_{(s^*,x^*,u^*)}\delta u(t) \\ \frac{\partial \delta x(z,t)}{\partial t} = \frac{\partial f_2(s,x)}{\partial s}_{(s^*,x^*)}\delta s(z,t) + \frac{\partial f_2(s,x)}{\partial x}_{(s^*,x^*)}\delta s(z,t) \\ 0 < z \le 1 \qquad t \ge 0 \end{cases}$$

Deriving $f_1$ with respect to $s$ , $x$ and $u$ at point $(s^*, x^*, u^*)$ we obtain

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$$\frac{\partial f_1}{\partial s}(s^*,x^*,u^*) = -u^*(t) - \frac{k_1 k_s \mu_{max} x^*}{(k_s+s^*)^2}$$
$$= -D - \frac{(\mu_{max}-D)^2}{\mu_{max} k_s}\left(s_a - \frac{Dk_s}{\mu_{max}-D}\right)$$

$$\frac{\partial f_1}{\partial x}(s^*,x^*,u^*) = -k_1 \frac{\mu_{max} s^*}{k_s+s^*} = -k_1 D$$

$$\frac{\partial f_1}{\partial u}(s^*,x^*,u^*) = s_a - s^* = s_a - \frac{Dk_s}{\mu_{max}-D}$$

Computing the derivatives of $f_2$ with respect to $s$ and $x$ at point $(s^*,x^*)$ gives

$$\frac{\partial f_2}{\partial s}(s^*,x^*) = \frac{k_s \mu_{max} x^*}{(k_s+s^*)^2} = \frac{(\mu_{max}-D)^2}{\mu_{max} k_1 k_s}\left(s_a - \frac{Dk_s}{\mu_{max}-D}\right)$$

$$\frac{\partial f_2}{\partial x}(s^*,x^*) = \frac{\mu_{max} s^*}{k_s+s^*} - D = 0$$

Then

$$\frac{ds}{dt} - \frac{ds^*}{dt} = \left[-D - \frac{(\mu_{max}-D)^2}{\mu_{max} k_s}\left(s_a - \frac{Dk_s}{\mu_{max}-D}\right)\right](s-s^*)$$
$$+\left[-k_1 D\right](x-x^*) + \left[s_a - \frac{Dk_s}{\mu_{max}-D}\right](u-u^*)$$

$$\frac{dx}{dt} - \frac{dx^*}{dt} = \left[\frac{(\mu_{max}-D)^2}{\mu_{max} k_1 k_s}\left(s_a - \frac{Dk_s}{\mu_{max}-D}\right)\right](s-s^*)$$

Let denote $S = s - s^*, X = x - x^*, v = u - u^*$. Then

$$\begin{cases} \dfrac{dS}{dt} = \left[-D - \dfrac{(\mu_{max}-D)^2}{\mu_{max} k_s}\left(s_a - \dfrac{Dk_s}{\mu_{max}-D}\right)\right].S \\ \quad -k_1 D.X + \left(s_a - \dfrac{Dk_s}{\mu_{max}-D}\right).v \\ \dfrac{dX}{dt} = \dfrac{(\mu_{max}-D)^2}{\mu_{max} k_1 k_s}\left(s_a - \dfrac{Dk_s}{\mu_{max}-D}\right). \\ 0 < z \le 1 \qquad t \ge 0 \end{cases} \quad (14)$$

Let $Y = (S, X)$ then (14) is writing as

$$\begin{cases} \dfrac{dY}{dt}(z,t) = AY(z,t) + Bv(t) & \text{in } ]0,1]\times]0,T[ \\ Y(z,0) = \left(s_0(z) - s^*, x_0(z) - x^*\right) & \text{in } ]0,1] \end{cases}$$

where the matrixes $A$ and $B$ are given by :

$$A = \begin{pmatrix} -D - \dfrac{(\mu_{max}-D)^2}{\mu_{max} k_s}\left(s_a - \dfrac{Dk_s}{\mu_{max}-D}\right) & -k\_1\, D \\ \dfrac{(\mu_{max}-D)^2}{\mu_{max} k_1 k_s}\left(s_a - \dfrac{Dk_s}{\mu_{max}-D}\right) & 0 \end{pmatrix}$$

$$, \quad {}^t B = \begin{pmatrix} s_a - \dfrac{Dk_s}{\mu_{max}-D} & 0 \end{pmatrix}$$

and $v(t) = u(t) - D$ represents the applied control on $\Omega = ]0,1]$.

The matrix $A$ generates a semi-group denoted by $(e^{At})_{t\ge 1}$ and the solution of $(15)$ when it is excited by the control $v$, at the final time $T$ , is given by:

$$Y_v(z,T) = e^{AT}Y^0(z) + \int_0^T e^{A(T-t)}Bv(t)\,dt \quad \text{with} \quad Y^0(z) = (s_0(z$$

Let $Y_1 = (S_1, X_1)$ and $Y_2 = (S_2, X_2)$ be two states such that $S_1 \le S_2$ et $X_1 \le X_2$.

Our goal is to determinate the feed flow rate in substrate $f(t)$ such that the solution $Y_v(.)$ at the outlet of the bioreactor be between $Y_1$ and $Y_2$ at time $T$ . More precisely, we are interested in the following minimization problem

$$\begin{cases} \inf \dfrac{1}{2}\| v \|^2 \\ v \in U_{ad} \end{cases} \quad (16)$$

Where

$$U_{ad} = \{\, v \in L^2(0,T,IR) \,|\, Y_1 \le Y_v(1,T) \le Y_2 \,\}$$

***D. Simulations***

For numerical simulations we consider the following values for the different parameters

$k_1 = 1$ $\quad D = 0.1h^{-1}$ $\quad k_s = 15gl^{-1}$ $\quad \mu_{max} = 2.h^{-1}$

$s_a = 50gl^{-1}$ $\quad x_0(z) = 0.2gl^{-1}$ $\quad s_0 = 100.gl^{-1}$ $\quad \Gamma = 1$

Applying the algorithm mentioned in the previous section, the simulations give:

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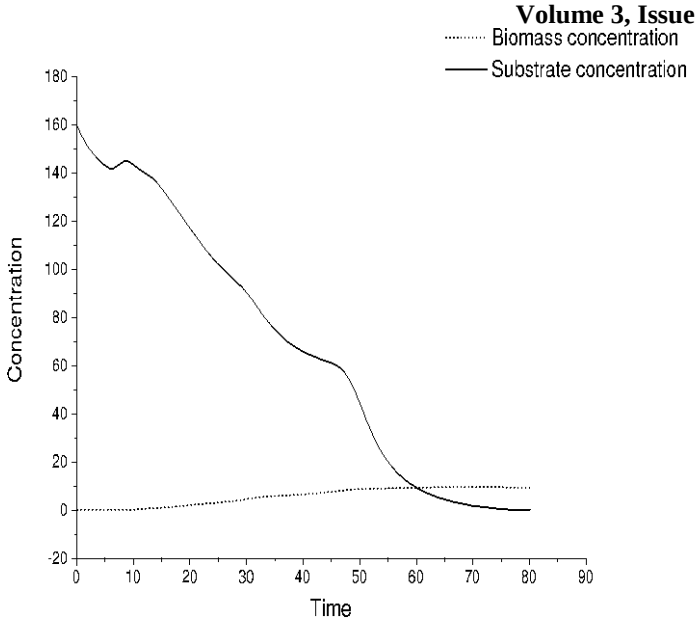


**Fig 2. Substrate evolution (continuous line) and the biomass evolution (dashed line) with**

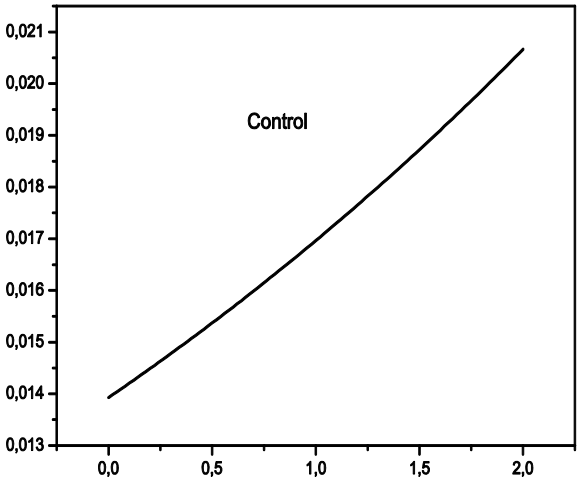


**Fig 3. Control function**

## V. CONCLUSION

In this work, the problem of optimal control of wastewater biological treatment was examined using tools of boundary constrained controllability [5]. The feed flow rate was considered as control variable. The optimal control was numerically obtained using Uzawa algorithm. The simulations illustrate the efficiency of used approach.

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