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A Common Fixed Point Theorem in Dislocated Metric Space

Surjeet Singh Chauhan[1] (Gonder) , Kiran Utreja[2]
Deptt. Of Applied Science and Humanities, Chandigarh University, Gharuan
Deptt. Of Applied Science and Humanities, GNIT, Mullana

Abstract: ***In this paper, we prove a common fixed point theorem for six mappings in dislocated metric space making use of the concept occasionally weakly compatible for six mappings.***

Mathematics Subject Classification: 47H10, 54H25

Keywords: **Dislocated metric spaces, occasionally weakly compatible, fixed point.**

I. INTRODUCTION

In 1922, a famous contraction principle known as Banach Contraction Principle [3] in metric space came into existence which became the active field of researchers for research to prove a number of fixed point theorems. In 2000, the concept of dislocated metrics was studied under the name of metric domains in the context of domain theory in [2] and notion of dislocated metric space came up in which self distance of a point need not be equal to zero. This concept was put forward by P.Hitzler and A.K.Seda [8] who also generalized the famous Banach Contraction Principle in this space. Mathematicians like C. T. Aageet al. [4], A. Isufati [1] ,K. Jha et al.[6], K. P. R. Rao et al.[7].established some important fixed point theorems in dislocated metric space with different conditions. Here we are proving a common fixed point theorem for six self maps using the concept of occasionally weak compatibility.

II. PRELIMINARIES

Def.2.1.[5] Let X be a non-empty set and let d: X xX→ [0,∞) be a function satisfying following conditions:

i. d(x, y) = d(y, x) ,
ii. d(x, y) = d(y, x) = 0 implies x = y,
iii. d(x, y)≤ d(x, z) +d(z, y) , for all x, y, z ∈ X.

Then d is called a dislocated metric (or d- metric) on X.

Def.2.2. [5] A sequence $\{x_n\}$ in a d- metric space (X, d) is called a Cauchy sequence if for given ∈>0, there corresponds n_0 ∈ N such that for all m, n≥ n_0, d (x_m, x_n) <∈.

Def.2.3.[5] Asequence $\{x_n\}$ in d-metric space converges with respect to d (or in d) if there exists x ∈ X such that d $(x_n$, x)→ 0as n→ ∞.

Def.2.4.[5] Ad-metric space (X, d) is said to be complete if every Cauchy sequence in it is convergent with respect to d.

Def.2.5. [5] Let (X, d) be d-metric space. A map T: X→ X is called contraction if there exists a number λ with 0≤ λ<1 such that d (Tx, Ty) ≤d(x, y).

Lemma.2.6. Let (X, d) be a d-metric space. If T: X→ X is a contraction function, then $\{T^n(x_0)\}$ is a Cauchy sequence for each x_0 ∈X.

Lemma.2.7. [5] Limits in a d- metric space are unique.

Def.2.9. Let f and g be two self mappings of a metric space (X, d), thenC (f, g) ={u∈ X: fu =gu}.

Def.2.10. Two self-maps are said to be occasionally weakly compatible if there exists at least one x ∈ X, for which f(x) = g(x) implies fg(x) = gf(x).

III. MAIN THEOREM

Theorem 3.1 Let A, B, P, Q, S and T be six self-maps of a complete d-metric space(X, d) satisfying:

i. P(X)⊆ST(X) and Q(X)⊆AB(X),
ii. C(P, AB) ≠ ∅ and C(Q, ST) = ∅,
iii. The pair (P,AB)and (Q, ST) are occasionally weakly compatible,
iv. d(Px, Qy)≤ ∅{min[d(ABx, STy), d(Px, ABx), d(Qy,STy)]}.

For all where x, y∈X, where$\emptyset: R^+ \to R^+$ is monotonically non-decreasing and $\sum_n^\infty \emptyset^n < \infty$ for all t>0.

Then P, AB, Qand ST have a unique common fixed point.

Proof: Letx_0 be any arbitrary point in X. Since P(X) ⊆ ST(X) and Q(X) ⊆AB(X).

Therefore define two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

y_{2n} = STx_{2n+1}= Px_{2n},

y_{2n+1} = ABx_{2n+2}= Qx_{2n+1} for n = 0, 1, 2…

Now d (y_{2n}, y_{2n+1}) =d (Px_{2n}, Qx_{2n+1})

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$\leq \emptyset\{\min [d(ABx_{2n}, STx_{2n+1}), d(Px_{2n}, ABx_{2n}), d(Qx_{2n+1}, STx_{2n+1})]\}$.

$= \emptyset\{\min [d(y_{2n-1}, y_{2n}), d(y_{2n}, y_{2n-1}), d(y_{2n+1}, y_{2n})]\}$.

$= \emptyset\{\min [d(y_{2n-1}, y_{2n}), d(y_{2n+1}, y_{2n})]\}$.

If $\min [d(y_{2n-1}, y_{2n}), d(y_{2n+1}, y_{2n})] = d(y_{2n+1}, y_{2n})$.

Then $d(y_{2n}, y_{2n+1}) \leq \emptyset\{d(y_{2n+1}, y_{2n})\} \leq d(y_{2n+1}, y_{2n})$.

Which is a contradiction, thus $\min [d(y_{2n-1}, y_{2n}), d(y_{2n+1}, y_{2n})] = d(y_{2n-1}, y_{2n})$.

Therefore we get $d(y_{2n}, y_{2n+1}) \leq d(y_{2n-1}, y_{2n})$.

That is $d(y_{2n}, y_{2n+1}) \leq \emptyset[d(y_{2n-1}, y_{2n})] \leq \emptyset^2[d(y_{2n-2}, y_{2n-1})] \leq \cdots \leq \emptyset^n d(y_0, y_1)$.

Now for n, m$\in$ N, n< m, we have

$d(y_n, y_m) = d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots\ldots + d(y_{m-1}, y_m)$

$\leq \emptyset^n[d(y_0, y_1)] + \emptyset^{n+1}[d(y_0, y_1)] + \ldots\ldots\ldots + \emptyset^{m-1}[d(y_0, y_1)]$.

$\leq \sum_{i=1}^{n} \emptyset^i[d(y_0, y_1)]$.

$\rightarrow 0$ as n, m$\rightarrow \infty$.

Hence $\{y_n\}$ is a Cauchy sequence in the dislocated metric space X.

Therefore there exists u $\in$X such that $\{y_n\}$ converges to u.

Thus $\{Px_{2n}\}$, $\{STx_{2n+1}\}$, $\{ABx_{2n+2}\}$ and $\{Qx_{2n+1}\}$ converge to u.

Since P(X)$\subseteq$ST(X), thus there exists z$\in$ X such that u = STz.

Now $d(u, Qz) = d(Px_{2n}, Qz) \leq \emptyset\{\min [d(ABx_{2n}, STz), d(Px_{2n}, ABx_{2n}), d(Qz, STz)]\}$.

$\leq \emptyset\{\min [d(u, u), d(u,u), d(Qz,u)]\}$.

Since d (u, u) $\leq$d (u, Qz) +d (Qz, u).

Thus d (u, Qz) $\leq \emptyset\{\min [2d(Qz,u), d(Qz,u)]\}$.

This implies d (u, Qz) $\leq \emptyset[d(u, Qz)]$.

That is d (u,Qz)$\leq$d (u, Qz) a contradiction.

Thus Qz = u.

Hence STz = Qz = u.

Thus C (ST, Q)$\neq \emptyset$.

Also Q(X) $\subseteq$AB(X), therefore there exists w $\in$ X such that u = ABw.

Now d (Pw, u) =d (Pw, Qx_{2n+1})

$\leq \emptyset\{\min [d(ABw, STx_{2n+1}), d(Pw, ABw), d(Qx_{2n+1}, STx_{2n+1})]\}$.

$\leq \emptyset\{\min [d(u, u), d(Pw,u), d(u,u)]\}$.

$\leq \emptyset[d(Pw,u)]$.

Thus d (Pw, u) $\leq$d (Pw,u) a contradiction.

Therefore Pw= ABw =u.

Thus C (P, AB) $\neq \emptyset$.

Thus we have STz = Qz = Pw= ABw =u.

As the pair (P, AB) occasionally weakly compatible andC (P, AB) $\neq \emptyset$ this implies that there exists w$\in$C (P, AB), such that PABw= ABPw

This implies Pu = ABu.

This implies u is coincidence point of P, AB.

Similarly the pair (Q, ST) is occasionally weakly compatible and C (Q, ST) =$\emptyset$, thus there exists v$\in$C (Q, ST) such that STQv= QSTv

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This implies STu= Qu.

This shows u is coincidence point of Q and ST.

Now to show that u is coincidence point of AB, ST, P and Q.

For this put x = u and y = x_{2n+1} in (IV), we get

d (Pu, Qx_{2n+1}) $\leq \emptyset$\{min [d(ABu, STx_{2n+1}), d(Pu, ABu), d(Qx_{2n+1},STx_{2n+1})]\}.

Take the limit as n$\rightarrow \infty$, we get

d (Pu, u) $\leq \emptyset$\{min [d(Pu,u), d(Pu, Pu), d(u,u)]\}.

As d (Pu, Pu)$\leq$d (Pu, u) +d (u, Pu).

Thus d (Pu, u) $\leq \emptyset$[d (Pu, u)]$\leq$d (Pu, u).

which is a contradiction.

Hence Pu = u.

But ABu = Pu.

Therefore ABu= Pu = u.

This shows u is coincidence point of AB and P.

Next to prove that u is also the coincidence point of Q and ST.

For this put x= x_{2n} and y = u in (IV), we get

d (Px_{2n}, Q u)$\leq$ $\emptyset$\{min[d(ABx_{2n}, ST u), d(Px_{2n}, ABx_{2n}), d(Q u,ST u)]\}.

Now take the limit as n$\rightarrow \infty$, we get

d(u, Q u)$\leq \emptyset$\{min[d(u, Q u), d(u,u), d(Q u,Q u)]\}.

Also as above d (Qu, Qu) $\leq$d (Qu, u) +d (Qu, u) .

Thus d (u, Qu)$\leq$ d(u, Qu) which is contradiction .

Hence Qu = u.

But Qu = STu.

Therefore Stu = Qu = u.

Thus we get Pu = Qu= ABu = STu = u.

This shows u is fixed point of P, AB, Q and ST.

IV. UNIQUENESS

Let u $\neq$ v be two common fixed points of the mappings P, AB, Q and ST. Then we have

d(u, v) = d(Pu, Qv)$\leq \emptyset$\{min[d(ABu, STv), d(Pu, ABu), d(Qv,STv)]\}.

$\leq \emptyset$\{min[d(u, v), d(u, u), d(v,v)]\}.

But d(u, u) $\leq$ d(u, v) +d(v, u) and d(v, v) $\leq$ d(v, u) +d(u, v).

Thus d (u, v) $\leq \emptyset[\, d(u,v)]$ $\leq$d (u, v), a contradiction.

Thus u = v. This proves the result.

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AUTHOR BIOGRAPHY

KiranUtreja, M.Sc. (MATHEMATICS), M.Phil. UGC and Ph.D. pursuing in fixed point theory is presently working as assistant professor in Guru Nanak Institute of Technology , Hemamajra, Ambala, Haryana, India having teaching experience of near about fourteen years with seven publications in different publications. For correspondence the email address iskiranutreja41@gmail.com

Dr. Surjeet Singh Chauhan, M.Sc. (MATHEMATICS), Ph.D. is presently working as professor in Deptt. Of Applied Science and Humanities, Chandigarh University, Gharuan, Chandigarh, India is having teaching experience of about fifteen years with twenty journals

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published in different publications. For correspondence the email address is surjeetschauhan@yahoo.com