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Effect of Couple Stresses on the Natural Convection of Heat and Mass Transfer in a Vertical Channel with Asymmetric Wall Temperature & Concentration

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Abstract-In the present study the effect of couple stresses and buoyancy ratio on natural convection of heat and mass transfer in a vertical channel with asymmetric wall temperature & concentration is considered. The closed form analytic solution for the important Characteristics of fluid flow, heat transfer, and mass transfer are obtained. The results are numerically computed and depicted graphically. The effect of increase in couple stress parameter is to decrease the velocity and there by decrease total heat ratio and total species rate. As couple stress parameter tends to infinity the fluid becomes Newtonian.

Index Terms— **Analytical solution, Couple stress Fluid, Heat and Mass transfer, Vertical channel.**

I. INTRODUCTION

The subject of convection flow with heat and mass transfer has been extensively studied due to its importance in chemical and nuclear industries. But in recent years, the study of heat transfer in vertical channel flows through a porous space has attracted the interest of many investigators in view of its applications in many engineering problem such as Storage of radioactive nuclear waste materials transfer, separation processes in chemical industries, Filtration, transpiration cooling, transport processes in aquifers, ground water pollution, etc. Also, the study of magnetic field with porous medium is very important both from theoretical as well as practical point of view; because most of natural phenomena of the fluid flow are connected with porous medium (For instance, filtration of fluids, underground water and oil, Reservoir and fluid through pipes).

The effect of natural convection is accumulative so it cannot be ignored even when the flow acceleration is small. The simplest physical model is a two dimensional mixed forced and free convection along a flat plate. Understanding of fundamental mechanism of this interaction can help to estimate more accurately the heat transfer rate and pumping power f or geometrics of practical interest in order to prevent unnecessary burn-out of heated surfaces. Mixed convection heat transfer of non-Newtonian fluids on a flat plate has been investigated using a modified power-law viscosity model by M.M Molla and L.S. Yao [1].

Ching-Yang Cheng [2] examines the effects of the vortex viscosity parameter and the buoyancy ratio on the fully developed natural convection of heat and mass transfer of a micro polar fluid in a vertical channel with asymmetric wall temperature and concentration. Bodoia and Osterle [3] examined the fully developed natural convection of heat transfer in a vertical parallel-plate channel with uniform, equal wall temperature channel with asymmetric wall temperature. Aung [4] and Miyatake and Fuji [5] have studied natural convection of heat transfer for fully developed flow between vertical parallel plates with asymmetric boundary condition. Nelson and Wood [6] examined the fully developed natural convection of heat transfer in a vertical parallel-plate channel with asymmetric boundary condition. Chamkha [6] studied fully developed free convection and of a micro polar fluid in a vertical channel. Sharma and. Thakur [7] have studied a layer of electrically conducting couple-stress fluid heated from below in porous medium in presence of magnetic field. Hire math and Patil [8] analyzed the effects of free convection currents on the oscillatory flow of polar fluids through a porous medium, which is bounded by a Vertical plane surface of constant temperature. The onset of double diffusive convection in a two component couple stress fluid layer with Soret and Dufour effects has been studied using both linear and non-linear stability analysis by Gaikwad et.al [9]. Shiva kumara [10] theoretically investigated Onset of convection in a layer of couple-stress fluid-saturated porous medium is investigated for different types of basic temperature gradients.

In the present study effect of couple stresses and buoyancy ratio on natural convection of heat and mass is analyzed. Analytical solutions are obtained and results are graphically depicted.

II. MATHEMATICAL FORMULATION

A steady fully developed laminar natural convection in a couple stress fluids between two vertical plates is considered. The vertical plates are separated by a distance b. The inlet temperature is T_0 and the inlet concentration is C_0. The inner surface of the left plate (i, e, y = 0) is kept at

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a constant temperature T_1 while the inner surface of the right plate (i, e., y = b) is kept at a constant temperature T_2. In addition, the concentration of certain constituent in the solution varies from C_1 on the inner surface of the left plate to C_2 on the inner surface of the right plate. Because the flow is fully developed, the transverse velocity is zero, and the flow depends only on the transverse coordinate y. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. Governing Equations are

$$\mu\frac{d^2u}{dy^2}-\eta\frac{d^4u}{dy^4}+\rho g\beta_t\left(T-T_0\right)+\rho g\beta_c\left(C-C_0\right)=0 \quad (1)$$

$$\frac{d^2T}{dy^2}=0 \quad (2)$$

$$\frac{d^2C}{dy^2}=0 \quad (3)$$

The above equation are subjected to the boundary conditions

$$u=0 \;\; u''=0, T=T_1, C=C_1 \quad \text{At} \;\; y=0 \quad (4)$$

$$u=0 \;\; u''=0, T=T_2, C=C_2 \;\; \text{At} \;\; y=b \quad (5)$$

The above equations and boundary conditions are non dimensionalised with

$$Y=\frac{y}{b}, U=\frac{ub\rho}{Gr\mu}, \;\; \theta=\frac{T-T_0}{T_1-T_0},$$

$$\phi=\frac{C-C_0}{C_1-C_0}, Gr=\beta_t g\left(T_1-T_0\right)b^3\frac{\rho^2}{\mu^2} \quad (6)$$

Where Gr is a Grashof number.

After non-dimensionalisation, equations (1)-(5) become

$$\frac{d^2U}{dY^2}-\alpha^2\frac{d^4U}{dY^4}+\theta+N\phi=0 \quad (7)$$

$$\frac{d^2\theta}{dY^2}=0 \quad (8)$$

$$\frac{d^2\phi}{dY^2}=0 \quad (9)$$

$$U=0, U''=0, \quad \theta=1, \phi=1 \;\; \text{at} \;\; y=0 \quad (10)$$

$$U=0, U''=0, \;\; \theta=m, \phi=n \;\; \text{at y=1} \quad (11)$$

Where, $N=\dfrac{\beta_c\left(C_1-C_0\right)}{\beta_t\left(T_1-T_0\right)}$ is the buoyancy ratio,

$\alpha^2=\dfrac{\eta^2}{\mu^2b^2}$ is the couple stress parameter,

$m=\dfrac{\left(T_2-T_0\right)}{\left(T_1-T_0\right)}$ is the wall temperature ratio and

$n=\dfrac{\left(C_2-C_0\right)}{\left(C_1-C_0\right)}$ is the wall concentration ratio,

Solving equations. (8) And (9) using boundary conditions given by equations (10) and (11) we get,

The dimensionless temperature and concentration as

$$\theta=\left(m-1\right)Y+1.$$

$$(12) \;\; \phi=\left(n-1\right)Y+1 \quad (13)$$

Substituting equations (12)- (13) in equation (7) the velocity is obtained as

$$U=C_2Y+C_3e^{aY}+C_4e^{-aY}-\frac{1}{a^2}\left(\frac{MY^3}{6}+\frac{N_0Y^2}{2}+\frac{MY+N_0}{a^2}\right) \quad (14)$$

where $C_2=\dfrac{1}{a^2}\left[M\left(\frac{1}{6}+\frac{1}{a^2}+\frac{1}{a^4}\right)+N_0\left(\frac{1}{2}+\frac{1}{a^2}-\frac{e^a}{a^4}+\frac{e^a}{a^4}\coth a-\frac{\coth a}{a^4}\right)\right]$

$$C_3=\frac{N_0}{a^4}-\frac{N_0\left(e^a-1\right)-M}{2a^4\sinh a},$$

$$C_4=\frac{N_0\left(e^a-1\right)-M}{2a^4\sinh a}, \quad a^2=\frac{1}{\alpha^2},$$

$$M=m+nN-1-N, \;\; N_0=N+1 \quad (15)$$

The dimensionless volume rate is given by

$$Q=\int_0^1 U\,dY \quad (16)$$

The dimensionless total heat rate added to the fluid can be expressed as $E=\int_0^1 U\theta\,dY$ (17)

Substituting equations (12) and (14) into equation (17) and integrating, the dimensionless total heat is obtained as

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$$E=C_2\left(\frac{2m+1}{6}\right)+C_3\left[\frac{1}{a}\left(me^{a}-1\right)+\frac{1}{a^2}\left(1-e^{a}\right)\left(m-1\right)\right]+C_4\left[\frac{1}{a}\left(1-me^{-a}\right)+\frac{1}{a^2}\left(1-e^{-a}\right)\left(m-1\right)\right]-\frac{1}{a^2}\left[M\left(\frac{1}{120}-\frac{1}{3a^2}\right)+Mm\left(\frac{1}{10}+\frac{1}{a^2}\right)+\frac{MN_0}{2}\left(\frac{1}{a^2}+\frac{1}{4}\right)\right]+\frac{N_0}{2}\left(\frac{1}{12}+\frac{1}{a^2}\right). \quad (18)$$

The dimensionless total species rate is given

by, $$\Phi=\int_0^1 U\phi\, dY \quad (19)$$

substituting equations (13) and (14) into equation (19) and integrating, we obtain the dimensionless total species added to the fluid as

$$\Phi=C_2\left(\frac{2n+1}{6}\right)+C_3\left[\frac{1}{a}\left(ne^{a}-1\right)+\frac{1}{a^2}\left(1-e^{a}\right)\left(n-1\right)\right]+C_4\left[\frac{1}{a}\left(1-ne^{-a}\right)+\frac{1}{a^2}\left(1-e^{-a}\right)\left(n-1\right)\right]-\frac{1}{a^2}\left[M\left(\frac{1}{120}-\frac{1}{3a^2}\right)+Mn\left(\frac{1}{10}+\frac{1}{a^2}\right)+\frac{MN_0}{2}\left(\frac{1}{a^2}+\frac{1}{4}\right)\right]+\frac{N_0}{2}\left(\frac{1}{12}+\frac{1}{a^2}\right). \quad (20)$$

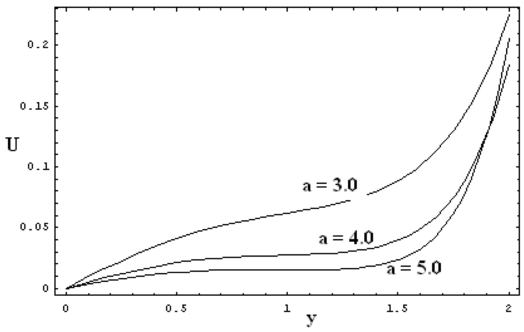

Figure 1. Velocity profile for different values of couple stress parameter

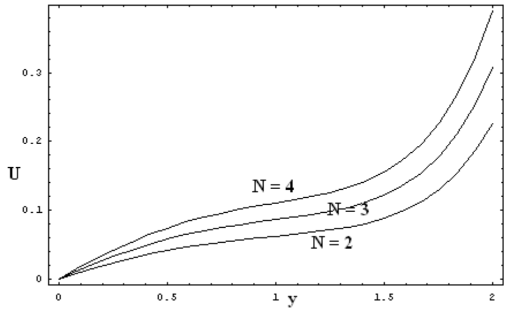

Figure 2. Velocity profile for different values of Bouyancy ratio

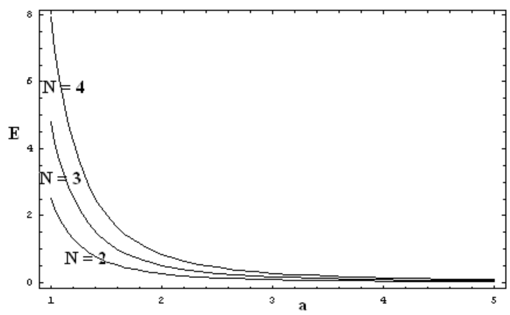

Figure 3. Plot of total heat ratio vs couple stress parameter 'a'

III. RESULTS AND DISCUSSION

The Velocity profile, dimensionless total heat rate and dimensionless total species rate are numerically calculated and graphically depicted.**Fig1.**Shows Velocity profile for different values of couple stress parameter. Velocity profile increases in y-direction. Velocity decreases with increasing couple stress parameter. The couple stress parameter is inversely proportional to spin of the particles. As spin increases velocity increases, when couple stress parameter a→∞ fluid becomes Newtonian. **Fig2.** Shows Velocity profile for different values of Buoyancy ratio. Velocity increases with buoyancy ratio N. **Fig3. S**hows plot of total heat rate against couple stress parameter for different values of buoyancy ratio. Total heat rate decreases with increase in couple stress parameter and same argument as in case of velocity holds.

Fig4. Shows plot of total heat rate against buoyancy ratio for different values couple stress parameter. The fig. Shows increasing curve for total heat rate. **Fig5.** Shows plot of total species rate against couple stress parameter**, Fig6.** Shows plot of total species rate against buoyancy ratio. Species also behaves in the same manner as heat.

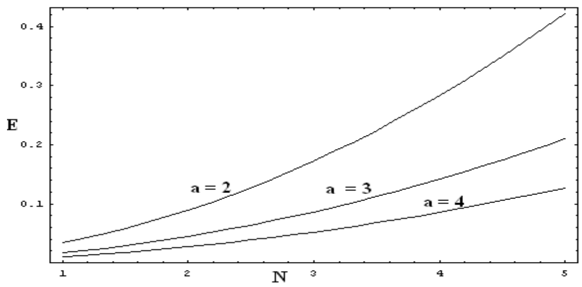

Figure 4. Plot of Total heat rate vs bouyancy parameter

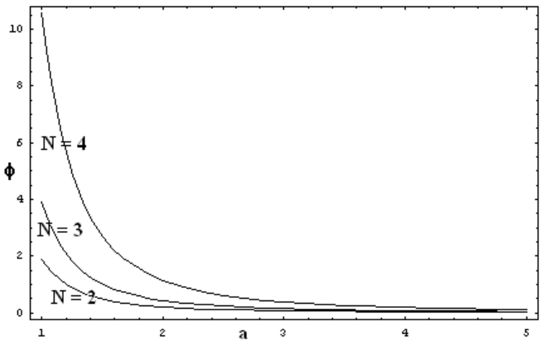

Figure 5. Plot of Total species rate vs couple stress parameter

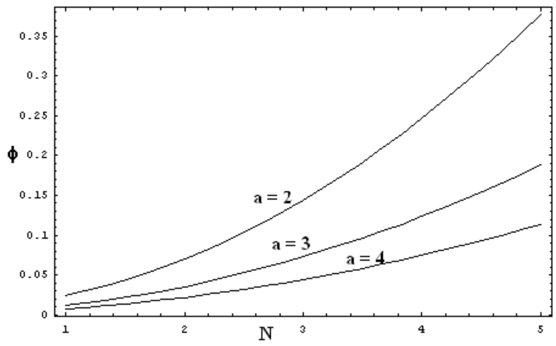

Figure 6. Plot of total species rate vs bouyancy ratio

IV. CONCLUSION

Velocity profile is parabolic. Velocity decreases with increase in couple stress parameter. Couple stress parameter is inversely proportional to the spin of the suspensions; hence increase in couple stress is characterized by decrease in velocity. As $a \to \infty$ fluid behaves as Newtonian. Velocity increases with increasing value of N_O, buoyancy ratio. Increase in buoyancy results in increase in spinning of the suspension, hence increase in velocity. Total heat shows a linear increase in profile with buoyancy ratio. Total heat decreases with increase in couple stress parameter.

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