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Complex Neuro Fuzzy System Using Complex Fuzzy Sets and Update the Parameters by PSO-GA and RLSE Method

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Abstract*— *The novelty of the complex fuzzy sets lies in the range of values its membership function may attain. In contrast to a traditional fuzzy membership function, this range is not limited to [0, 1], but extended to the unit circle in the complex plane. Thus, the complex fuzzy set provides a mathematical framework for describing membership in a set in terms of a complex number. Based on the property of complex-valued membership, Complex fuzzy sets can be used to design a neural fuzzy system so that the Complex Neuro Fuzzy System (CNFS) can have excellent adaptive ability. The Hybrid PSO with GA is a multi-swarm-based optimization method, proposed by us, and it is used to adjust the premise parameters and the RLSE method is used to update the consequent parameters of the CNFS.

***Index Terms*— Complex fuzzy set (CFS), Complex neuro-fuzzy system (CNFS), Hybrid particle swarm optimization with Genetic Algorithm (Hybrid PSO with GA), Recursive least square estimator (RLSE).**

I. INTRODUCTION

Complex fuzzy set (CFS) [8]-[9] is a new development in the theory of fuzzy systems. The concept of CFS is an extension of fuzzy set, by which the membership for each element of a complex fuzzy set is extended to complex-valued state. In a complex fuzzy set, membership values are complex numbers in the unit disc of the complex plane [8]-[9]. Although the introductory theory of the CFS has been presented [8], the research on complex fuzzy system designs and applications using the concept of CFS is found rarely. Since the seminal paper in 1965 by Zadeh proposed *Fuzzy Sets* [10], a huge amount of literature has appeared on different aspects of fuzzy sets and their applications. Ramot et al. [8] proposed an important extension of these ideas, the *Complex Fuzzy Sets*, where the membership function μ instead of being a real valued function with the range [0,1] is replaced by a complex-valued function of the form

$$r_S(x).e^{j\omega_S(x)} \quad ; \quad j=\sqrt{-1}$$

Where $r_S(x)$ and $\omega_S(x)$ are both real valued giving the range as the unit circle. However, this concept is different from fuzzy complex number introduced and discussed by Buckley [1]-[4] and Zhang [5]-[7]. Essentially as explained in [8] this still retains the characterization of the uncertainty through the amplitude of the grade of membership having a value in the range of [0,1] whilst adding the membership phase captured by fuzzy sets . As explained in Ramot et al [8], the key feature of complex fuzzy sets is the presence of phase and its membership. This gives those complex fuzzy sets wavelike properties which could result in constructive and destructive interference depending on the phase value. Thus property distinguishes these complex fuzzy sets from conventional fuzzy sets, fuzzy complex sets, and type 2 fuzzy sets [12] (a brief comparison of them in Appendix). Several examples are given in [8] which demonstrate the utility of these complex fuzzy sets. Complex neuro-fuzzy system (CNFS) [14] is an same architecture of Adaptive Neuro Fuzzy Inference System (ANFIS) [13]. But CNFS using the theory of complex fuzzy set (CFS) to achieve high prediction accuracy for the problem of time series forecasting. For the proposed CNFS, we devise a new learning method, which combines a Hybrid Particle swarm optimization with Genetic Algorithm [15] (Hybrid PSO with GA) and Recursive Least Square (RLSE) Algorithm. Hybrid PSO with GA method used to update the premise parameters and RLSE used to update the consequent parameters. Complex fuzzy set defined in section II. Complex neuro fuzzy system (CNFS) is discussed in section III. Section IV describes a Hybrid PSO with Genetic Algorithm approach and RLSE method in CNFS. Finally, the paper is concluded.

II. THEORY OF COMPLEX FUZZY SET

A complex fuzzy set (CFS) may be visually represented as in Fig.1. In this three-dimensional (3-D) graph, the complex plane R XI is placed at right angles to the universe of discourse U (in this case, the real line),and the unit disc D is projected along U. A complex fuzzy set will then consist of a trajectory within this cylinder. Since μ is a function, this trajectory does not intersect with itself, nor can it "backtrack"; for any x ∈ U ,a disc centered at x and perpendicular to U may intersect the trajectory of μ exactly once.

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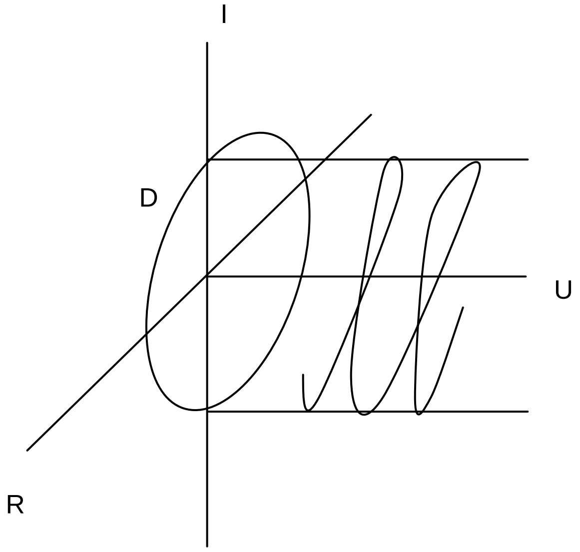
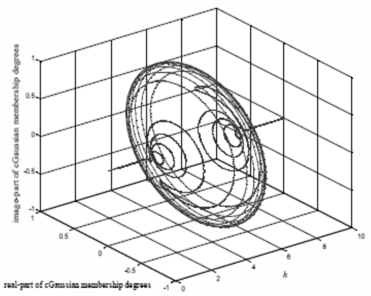


Fig.1: Complex Fuzzy Set [11]

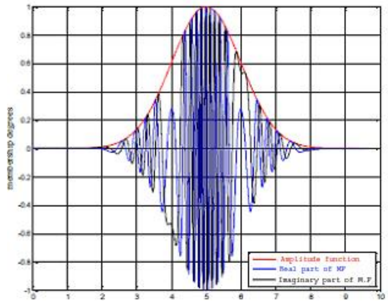
The theory of complex fuzzy set (CFS) can provide a new development for fuzzy system research and application [8]-[9]. The membership function to characterize a CFS consists of an amplitude function and a phase function. In other words, the membership of a CFS is in the two-dimensional complex-valued unit disc space, instead of in the one-dimensional real-valued unit interval space. Thus, CFS can be much richer in membership description than traditional fuzzy set. Assume there is a complex fuzzy set *S* whose membership function $\mu_S(h)$ is given as follows.

$$\begin{aligned}\mu_S(h) &= r_S(h)e^{j\omega_S(h)}\\ &= \mathrm{Re}(\mu_S(h)) + j\,\mathrm{Im}(\mu_S(h)) \qquad (2.1)\\ &= r_S(h)\cos(\omega_S(h)) + jr_S(h)\sin(\omega_S(h))\end{aligned}$$

where $j = \sqrt{-1}$, *h* is the base variable for the complex fuzzy set, $r_s(h)$ is the amplitude function of the complex membership, $\omega_s(h)$ is the phase function. The property of sinusoidal waves appears obviously in the definition of complex fuzzy set. In the case that $\omega_s(h)$ equals to 0, a traditional fuzzy set is regarded as a special case of a complex fuzzy set. A novel Gaussian-type complex fuzzy set is shown in Fig. 2.



(a)



(b)

Fig.2 Illustration of Gaussian-Type Complex Fuzzy set. [14]

(a) 3-D view with the coordinates of base variable, real-part and imaginary-part membership.
(b) Amplitude membership and imaginary-part membership vs base variable.

The Gaussian-type complex fuzzy set, denoted as *cGaussian*(h, m, σ, λ), is designed as follows.

$$cGaussian(h,m,\sigma,\lambda)=r_s(h, m, \sigma)\, e^{(j\omega_s(h,m,\sigma,\lambda))} \qquad (2.2)$$

$$r_s(h, m, \sigma) = Gaussian(h, m, \sigma) = e^{\left[-05\left(\frac{h-m}{\sigma}\right)^2\right]} \qquad (2.3)$$

$$\omega_s(h, m, \sigma, \lambda) = -e^{\left[-05\left(\frac{h-m}{\sigma}\right)^2\right]} * \left(\frac{h-m}{\sigma^2}\right) * \lambda \qquad (2.4)$$

In (2.2) to (2.4), *h* is the base variable and $\{ m, \sigma, \lambda \}$ are the parameters of mean, spread and phase frequency factor for the complex fuzzy set.

III. COMPLEX NEURO FUZZY SYSTEM [14]

We devise a Gaussian complex fuzzy set (GCFS) for the design of the premises of CNFS. The general form of a GCFS is characterized below.

$$\mathrm{GCFS}(h,m,\sigma)=\exp\left[-\frac{(h-m)^2}{2\sigma^2}\right] + j\,\frac{-(h-m)}{\sigma^2}\left[-\frac{(h-m)^2}{2\sigma^2}\right]$$
(3.1)

Where $j=\sqrt{-1}$ m and σ are the mean and the spread of the GCFS, respectively. Suppose that we have a CNFS that consists of K first –order Takage-Sugeno(T-S) fuzzy rules, given as follows.

Rule i: IF (x_1 is $A_1^{(i)}(h_1)$) and (x_2 is $A_2^{(i)}(h_2)$) . . . and (x_M is $A_M^{(i)}(h_M)$)

Then $z^{(i)}=a_0^{(i)}+\sum_{j=1}^{M} a_j^{(i)} h_j$

For i=1,2…..K, where i is the index for the i th fuzzy rule; M is the number of inputs; x_j is the jth input linguistic variable for j=1,2,….M; h_j is the j th base variable; $A_j^{(i)}(h_j)$ is the jth premise, which is defined by a GCFS in (3.1); $z^{(i)}$ is the nominal output; [$a_j^{(i)}$,j=0,1,2….M] are consequent parameters. The complex fuzzy inference process can be cast into neural network structure to become a CNFS with six layers, specified below.

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Layer 0: The layer is called the input layer, which receives the inputs and transmits them to the next layer directly. The input vector is given as follows.

$$H(t)=[h_1(t),h_2(t),\ldots,h_M(t)]^T \qquad (3.2)$$

Layer 1: The layer is called the fuzzy-set layer. Nodes in the layer are used to represent the complex fuzzy sets for the premise part of the CNFS and to calculate the membership degrees.

Layer 2: This layer is for the firing-strengths. The firing strength of the i-th rule is calculated as follows.

$$\beta^i(t) = \mu_1^i(h_1(t)) * \mu_2^i(h_2(t)) * \ldots * \mu_M^i(h_M(t)) =$$

$$= \wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap \ldots A_M^i}) \qquad (3.3)$$

i=1,2….K, where min operator is used for the t-norm calculation of the firing strength r_j^i is the amplitude of complex membership degree for the j-th fuzzy set of the i-th rule.

Layer3. This layer is for the normalization of the firing strengths. The normalized firing strength for the i-th rule is represented as follows.

$$\lambda^i(t) = \frac{\beta^i(t)}{\sum_i^K \beta^i(t)} = \frac{\wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap \ldots A_M^i})}{\sum_i^K (\wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap \ldots A_M^i}))} \qquad (3.4)$$

Layer 4. The layer is for normalized consequents. The normalized consequent of the i-th rule is represented as follows.

$$\zeta^i(t) = \lambda^i(t)\, X\, z^i(t)$$

$$= \lambda^i(t) X (a_0^i + \sum_{j=1}^{M} a_j^i h_j(t))$$

$$\frac{\wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap A_2^i \ldots A_M^i})}{\sum_{i=1}^{K}(\wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap A_2^i \ldots A_M^i}))} X$$

$$(a_0^i + \sum_{j=1}^{M} a_j^i h_j(t)) \qquad (3.5)$$

Layer 5: This layer is called the output layer. The normalized consequents from Layer4 are congregated in the layer to produce the CNFS output, given as follows.

$$\zeta(t) = \sum_{i=1}^{K} \zeta^i(t) = \sum_{i=1}^{k} \lambda^i(t)\, X\, z^i(t) =$$

$$\sum_{i=1}^{k} \frac{\wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap \ldots A_M^i})}{\sum_{i=1}^{K}(\wedge_{j=1}^{M} r_j^i(h_j(t)) \exp(j\omega_{A_1^i \cap \ldots A_M^i}))} X (a_0^i + \sum_{j=1}^{M} a_j^i h_j(t)) \qquad (3.6)$$

Generally the output of the CNFS is represented as follows.

$$\zeta(t) = \zeta_{Re}(t) + j\zeta_{Im}(t)$$

$$= |\zeta(t)| \text{ x } \exp(j\omega_\zeta)$$

$$= |\zeta(t)| X \cos(\omega_\zeta) + j |\zeta(t)| X \sin(\omega_\zeta) \qquad (3.7)$$

Where $\zeta_{Re}(t)$ is the real part, $\zeta_{Im}(t)$ is the imaginary part of the output for the CNFS,

$$\zeta(t) = \sqrt{(\zeta_{Re}(t))^2 + (\zeta_{Im}(t))^2} \qquad (3.8)$$

$$\omega_\xi = \tan^{-1}\left(\frac{\zeta_{Im}}{\zeta_{Re}}\right) \qquad (3.9)$$

The absolute value of the complex output is given in (3.8), and the phase of the complex output is expressed in (3.9). Based on (3.6), the complex inference system can be viewed as a complex function system, expressed as follows

$$\zeta(t) = F(H(t),W) = F_{Re}(H(t),W) + jF_{Im}(H(t),W) \qquad (3.10)$$

Where $F_{Re}(.)$ is the real part of the CNFS output, $F_{Im}(.)$ is the imaginary part of the output, H(t) is the input vector to the CNFS, W denotes the parameter set of the CNFS. The parameter set W can be divided into two subsets, which are the premise-part subset and the consequent-part subset, denoted as W_{If} and W_{Then}, respectively.

IV. HYBRID PSO WITH GA – RLSE LEARNING FOR CNFS

PARTICLE SWARM OPTIMIZATION:

Swarm Intelligence (SI) is an innovative distributed intelligent paradigm for solving optimization problems that originally took its inspiration from the biological examples by swarming, flocking and herding phenomena in vertebrates. Particle Swarm Optimization (PSO) incorporates swarming behaviors observed in flocks of birds, schools of fish, or swarms of bees, and even human social behavior, from which the idea is emerged (Kennedy, 2001) (Clerc , 2002), (Parsopoulos, 2004). PSO is a population-based optimization tool, which could be implemented and applied easily to solve various function optimization problems. As an algorithm, the main strength of PSO is its fast convergence, which compares favorably with many global optimization algorithms like Genetic Algorithms (GA) (Goldberg, 1989) Simulated Annealing (SA) (Orosz, 2002), (Triki, 2005) and other global optimization algorithms. For applying PSO successfully, one of the key issues is finding how to map the problem solution into the PSO particle, which directly affects its feasibility and performance. The original PSO formulae define each particle as potential solution to a problem in D-dimensional space. The position of particle " i " is represented as

$$X_i=(x_{i1},x_{i2},\ldots\ldots x_{iD})$$

Each particle also maintains a memory of its previous best position, represented as

$$P_i=(p_{i1},p_{i2},\ldots\ldots p_{iD})$$

A particle in a swarm is moving; hence, it has a velocity, which can be represented as

$$V_i=(v_{i1},v_{i2},\ldots\ldots v_{iD})$$

Each particle knows its best value so far(pbest) and its position. Moreover, each particle knows the best value so far in the group (gbest) among pbests. This information is analogy of knowledge of how the other particles around them have performed. Each particle tries to modify its position using the following information:

1. The distance between the current position and pbest

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2. The distance between the current position and gbest. This modification can be represented by the concept of velocity. Velocity of each agent can be modified by the following equation (4.1) in inertia weight approach (IWA)

$$v_{id} = w^* v_{id}+c_1*r_1*(P_{id} -X_{id})+c_2*r_2*(P_{gd} - X_{id}) \quad (4.1)$$

Where, v_{id} – velocity of particle
x_{id} – current position of particle
w – inertial factor
c_1- determine the relative influence of the cognitive component
c_2- determine the relative influence of the social component
P_{id}- pbest of particle i
P_{gd} – gbest of the group
r_1,r_2 – random numbers

Where w is called as the inertia factor which controls the influence of previous velocity on the new velocity, r_1 and r_2 are the random numbers, which are used to maintain the diversity of the population, and are uniformly distributed in the interval [0,1]. C_1 is a postive constance, called as coefficient of the self-recognition component, C_2 is a positive constant, called as coefficient of the social component. From equation (4.1), a particle decides where to move next, considering its own experience, which is the memory of its best past position, and the experience of its most successful particle in the swarm. In the particle swarm model, the particle searches the solutions in the problem space with a range [-s,s].

V. HYBRID PSO WITH GA – RLSE LEARNING FOR CNFS

Hybrid PSO with GA is different from existed version of PSO [15]. The drawback of PSO is that the swarm may prematurely converge. The underlying principle behind this problem is that, for the global best PSO, particles converge to a single point, which is on the line between the global best and the personal best positions. This point is not guaranteed for a local optimum (Van den Bergh and Engelbrecht 2004). Another reason for this problem is the fast rate of information flow between particles, resulting in the creation of similar particles with a loss in diversity that increases the possibility of being trapped in local optima. A further drawback is that stochastic approaches have problem-dependent performance. This dependency usually results from the parameter settings in each algorithm. The different parameter settings for a stochastic search algorithm result in high performance variances. In general, no single parameter setting can be applied to all problems. Increasing the inertia weight (*w*) will increase the speed of the particles resulting in more exploration (global search) and less exploitation (local search) or on the other hand, reducing the inertia weight will decrease the speed of the particles resulting in more exploitation and less exploration. Thus finding the best value for the parameter is not an easy task and it may differ from one problem to another. Therefore, from the above, it can be concluded that the PSO performance is problem-dependent. The problem-dependent performance can be addressed through hybrid mechanism. It combines different approaches to be benefited from the advantages of each approach. To overcome the limitations of PSO, hybrid PSO algorithms with GA are proposed for updating premise parameters and RLSE for updating consequent parameters. The main problem with PSO is that it prematurely converges (Van den Bergh and Engelbrecht 2004) to stable point, which is not necessarily maximum. To prevent the occurrence, position update of the global best particles is changed. The position update is done through some hybrid mechanism of GA. The idea behind GA is due to its genetic operators crossover and mutation. By applying crossover operation, information can be swapped between two particles to have the ability to fly to the new search area. The purpose of applying mutation to PSO is to increase the diversity of the population and the ability to have the PSO to avoid the local maxima. There are three different hybrid approaches are proposed

1 PSO-GA (Type 1): The *gbest* particle position does not change its position over some designated time steps; the crossover operation is performed on *gbest* particle with chromosome of GA. In this model both PSO and GA are run in parallel.

2 PSO-GA (Type 2) : The stagnated *pbest* particles are change their positions by mutation operator of GA

3 PSO-GA (Type 3): In this model the initial population of PSO is assigned by solution of GA. The total numbers of iterations are equally shared by GA and PSO. First half of the iterations are run by GA and the solutions are given as initial population of PSO. Remaining iterations are run by PSO.

VI. RLSE METHOD

The RLSE [16] is used for the identification of the parameters of consequent part. For a general least-squares estimation problem, the output of a linear model, y, is specified by the linearly parameterized expression, given as follows.

$$y=\Theta_1 f_1(u) + \Theta_2 f_2(u)+\ldots\ldots.+ \Theta_m f_m(u) \quad (4.2)$$

where u is the model's input, $f_i(.)$ is known function of u and Θ_i, i=1,2,….m represents unknown parameters to be estimated. Here Θ_i can be viewed as the consequent parameters of the proposed T-S fuzzy approximator.

To estimate the unknown parameters { Θ_i , i=1,2,….m} for a unknown target system (or function), a set of input-output data pairs are used as training data, denoted as follows.

$$TD=\{(u_i,y_i),\ i=1,2,\ldots\ldots N\} \quad (4.3)$$

Substituting data pairs into (4.2), a set of N linear equations are given as follows.

$$\begin{aligned} f_1(u_1)\Theta_1+ f_2(u_1)\Theta_2+\ldots\ldots\ldots\ldots\ldots.+ f_m(u_1)\Theta_m&=y_1 \\ f_1(u_2)\Theta_1+ f_2(u_2)\Theta_2+\ldots\ldots\ldots\ldots\ldots.+ f_m(u_2)\Theta_m&=y_2 \quad (4.4) \\ \vdots \qquad \vdots \qquad\qquad\qquad \ddots \qquad \ddots \\ f_1(u_N)\Theta_1+ f_2(u_N)\Theta_2+\ldots\ldots\ldots\ldots\ldots.+ f_m(u_N)\Theta_m&=y_N \end{aligned}$$

The optimal estimation for Θ can be calculated using the following RLSE equation.

$$P_{k+1} = P_k - \frac{P_k b_{k+1} b_{k+1}^T P_k}{1 + b_{k+1}^T P_k b_{k+1}} \quad (4.5)$$

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$$\Theta_{k+1}= \Theta_k+P_{k+1}b_{k+1}(y_{k+1}-b_{k+1}^T \Theta_k) \quad (4.6)$$

k=0,1,….,N-1, where $[b^T_k, y_k]$ in the k-th row of [A,y]. To start the RLSE algorithm in (4.2), we need to select the initial values for θ_0 and P_0 is given as follows.

$$P_0=\alpha I$$

Where α is a large value and I is the identity matrix, and θ_0 is initially set to zeros. For the training of the proposed CNFS, the hybrid PSO with GA - RLSE learning method is applied to update the premise parameters and the consequent parameters respectively. For parameter learning, the proposed CNFS predictor is trained by the PSO with Ga and RLSE learning method, where the PSO with GA is used to update the premise parameters and the RLSE is used to adjust the consequent parameters. The training procedure for the proposed PSO with GA and RLSE method is given as follow.

Step1: Collect training data. Some portion of the data is used for training , and the rest is for Testing.

Step2: Update the premise parameters by the PSO with GA in 4.1 and use PSO-GA type1, type2 or type3.

Step3: Update the consequent parameters by the RLSE in (4.5) and (4.6), in which the row vector b and the vector θ are arranged as follows.

$$b_{k+1}=[bb^1(k+1) \quad bb^2(k+1) \quad \ldots bb^k(k+1)] \quad (4.7)$$

$$bb^i(k+1) = [\lambda^i \quad h_1(k+1)\lambda^i \quad \ldots h_M(k+1)\lambda^i] \quad (4.8)$$

$$\theta_k=[\tau_k^1 \quad \tau_k^2 \quad \ldots \tau_k^k] \quad (4.9)$$

$$\tau_k^i = [a_0^i(k) \quad a_1^i(k) \quad \ldots a_M^i(k) \quad (4.10)$$

Step4: Calculate the CNFS output in (3.10)

Step5: Calculate the cost in MSE defined below. Note that the time series forecasting problem in real valued domain, only the real part of the CNFS output is involved in MSE, $MSE=\frac{1}{N}\sum_{t=1}^{N}(e(t))^2$

$$MSE = \frac{1}{N}\sum_{t=1}^{N}(y(t) - Re(\zeta(t))^2 \quad (4.11)$$

Step6: Compare the cost in MSE defined below. Update pbest and gbest in the multiple swarms. If stopping criteria satisfied, gbest is the optimal premise Parameters for the CNFS and stop. Otherwise, go back to step2 and continue the procedure.

VI. CONCLUSION

The hybrid PSO with GA and RLSE learning method has been applied to the proposed CNFS to adapt its system parameters. The system parameters are divided into two subsets to make easier the learning process for the optimal solution to application performance. The two subsets are the premise set of parameters and the consequent set of parameters. The well-known PSO with GA is used to update the premise subset of parameters and the RLSE is for the consequent subset of parameters. This hybrid learning method is very efficient to find the optimal (or near optimal) solution for the CNFS in application performance.

VII. FUTURE WORK

We have planned to apply our proposed method in Prediction of Time series forecasting [14] such as Star brightness time series, Sea wave heights, Share market index etc. in experimentally.

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